

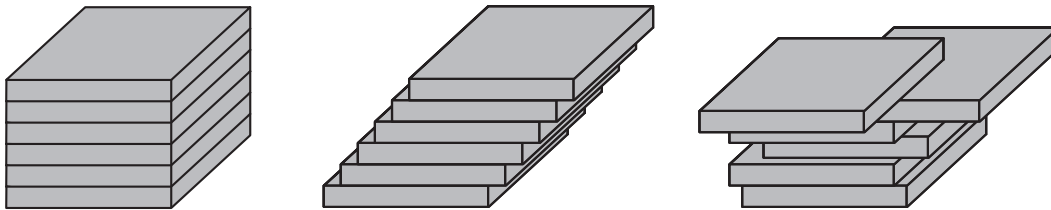
Volumes of Cylinders, Pyramids, and Cones

Key Concepts

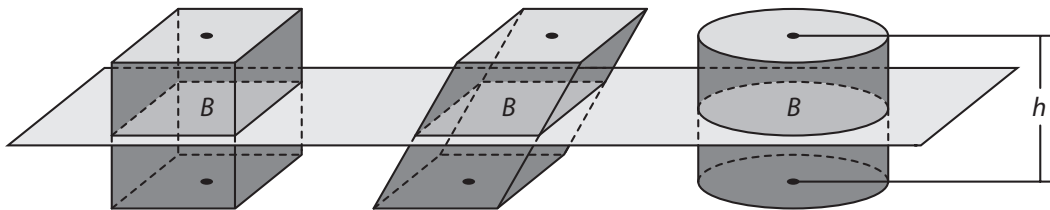
- The formula for finding the volume of a prism is $V = length \cdot width \cdot height$. This can also be shown as $V = area\ of\ base \cdot height$.
- Remember to use cubic units or volume measures when calculating volume. Some examples are cubic feet (ft^3), cubic meters (m^3), liters (L), and gallons (gal).

Cylinders

- A cylinder has two bases that are parallel. This is also true of a prism.
- Bonaventura Cavalieri, an Italian mathematician, formulated **Cavalieri's Principle**. This principle states that the volumes of two objects are equal if the areas of their corresponding cross sections are in all cases equal.
- This principle is illustrated by the diagram below. A rectangular prism has been sliced into six pieces and is shown in three different ways.

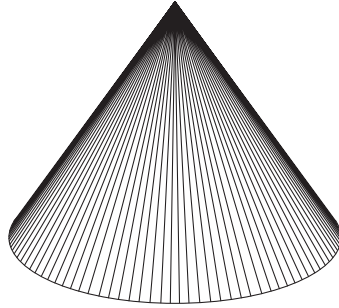


- The six pieces maintain their same volume regardless of how they are moved.
- Cavalieri's Principle describes how each piece is a thin slice in the plane of the prism.
- If each thin slice in each object has the same area, then the volumes of the objects are the same.
- The following diagram shows a prism, a prism at an oblique angle, and a cylinder.



Cones

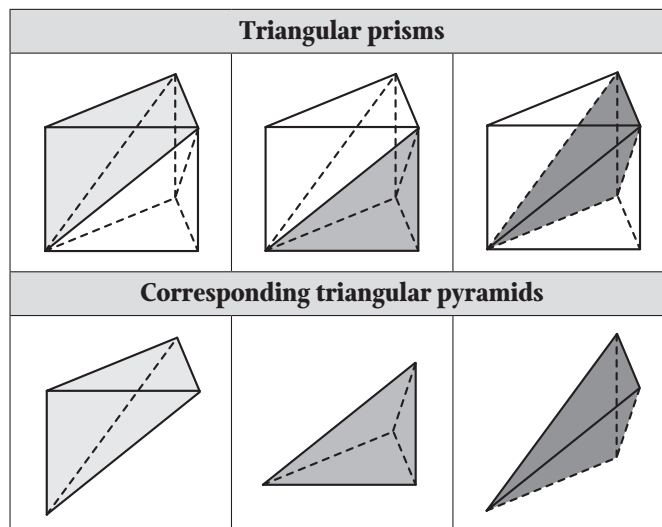
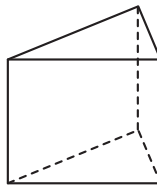
- A **cone** is a solid or hollow object that tapers from a circular base to a point.
- A cone and a pyramid use the same formula for finding volume.
- This can be seen by increasing the number of sides of a pyramid.
- The limit approaches that of being a cone.
- A pyramid with 100 sides follows. With such a large number of sides, it looks like a cone.



- The formula for the volume of a cone is $V_{\text{cone}} = \frac{1}{3}\pi r^2 \cdot h$.
- Cavalieri's Principle shows how pyramids and cones have the same volume.
- The diagram that follows shows cross sections of areas with the same planes.
- Each object has the same area at each cross section.

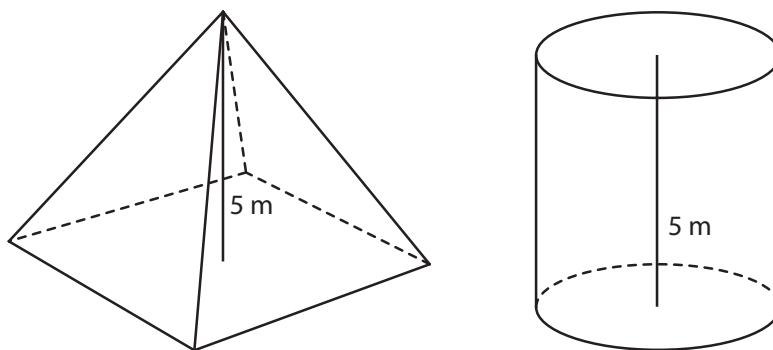
Pyramids

- A **pyramid** is a solid or hollow polyhedron object that has three or more triangular faces that converge at a single vertex at the top; the base may be any polygon.
- A **polyhedron** is a three-dimensional object that has faces made of polygons.
- A triangular prism can be cut into three equal triangular pyramids.



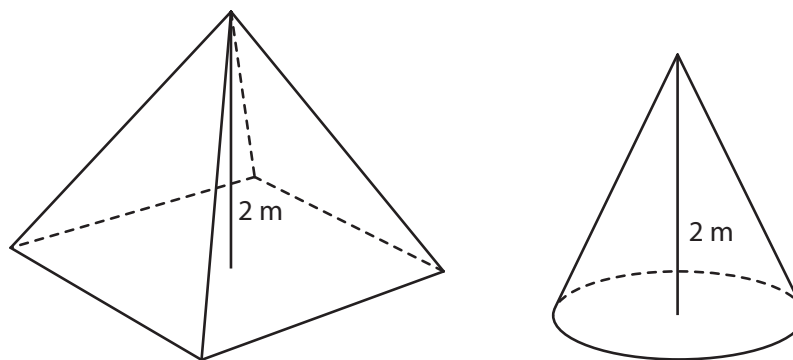
Example 1

Find the dimensions for a cylinder that has the same volume as a square prism with a base area of 9 square meters. The cylinder and the square prism should both have heights of 5 meters.



Example 3

Find the dimensions for a cone that has the same volume as a pyramid of the same height as the cone. Both the cone and the pyramid have a height of 2 meters. The volume of the pyramid is 3 cubic meters. A cone and a pyramid both taper to a point or vertex at the top. The “slant” of the taper is linear, meaning it is a straight line. The dimensions of both the cone and the pyramid change at a constant rate from base to tip.



Example 4

A jet fuel storage tank near a large airport is a cylinder that has a radius of 12.5 meters and a height of 15.854 meters. How many gallons of jet fuel will the tank hold? There are 264.172 gallons in 1 cubic meter.

Example 5

A new art museum is being built in the shape of a square pyramid. The height will be 50 meters. The art museum needs 86,400 cubic meters of space inside. What should be the side lengths of the base of the pyramid?

Practice 3.5.2: Volumes of Cylinders, Pyramids, and Cones

Use your knowledge of volume to complete each problem.

1. A gasoline fuel storage tank at an oil refinery is a cylinder with a radius of 20 meters and a height of 10 meters. How many gallons of gasoline will the tank hold? There are 264.172 gallons in 1 cubic meter.
2. A storage container is a cylinder with a height of 28.2 cm and a radius of 5 cm. How many liters of water will the storage container hold? 1 liter = 1000 cm³.
3. A company makes candles in the shape of cones. Their best-selling candle has a height of 6 inches and a circumference of 12 inches. What volume of wax is needed to make 1 candle?
4. For problem 3 above, what would the base side lengths be for a square pyramid candle that has the same volume and height as the cone?
5. In July 2012, an ice cream company in England set a new world's record for the largest ice cream cone ever made. The total height was 13 feet including the ice cream on top. The cone itself was approximately 9 feet tall and had a diameter of about 3.5 feet. How many gallons of ice cream were needed to fill just the cone part (not including the ice cream on top)?
1 cubic foot = 7.40852 gallons.