Unit 2 Inverse Trig Functions Task

Part I

1. Graph $f(\theta) = \sin\theta$ and the line y = ½ .

   a. How many times do these functions intersect between -2π and 2π?

   b. How is this graph related to finding the solution to ½ = sin θ?

   c. If the domain is not limited, how many solutions exist to the equation ½ = sin θ?

   d. Would this be true for the other trigonometric functions? Explain.

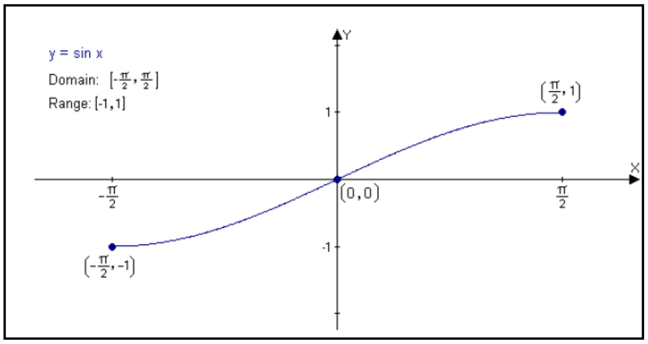
2. What happens to the axes and coordinates of a function when you reflect it over the line y = x?

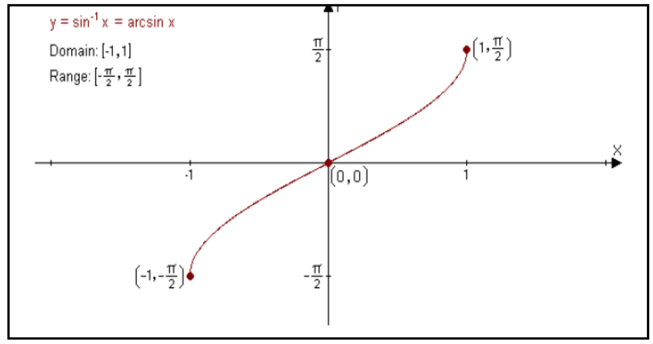
   a. Sketch a graph of $f(\theta) = \cos\theta$ and its reflection over the line y = x.

   b. How many times does the line x= ½ intersect the reflection of cos x?

3. Sketch the inverse of $f(\theta) = \tan\theta$ and determine if it is a function.

4. Look at this graph of $f(\theta) = \sin\theta$ with domain $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and its inverse.

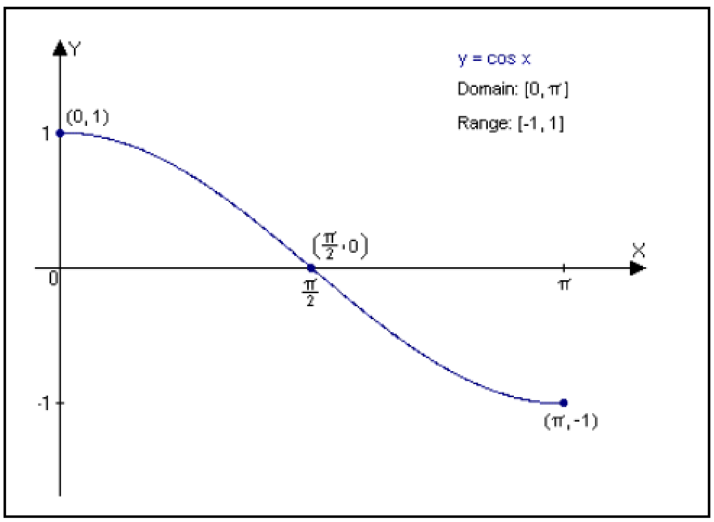


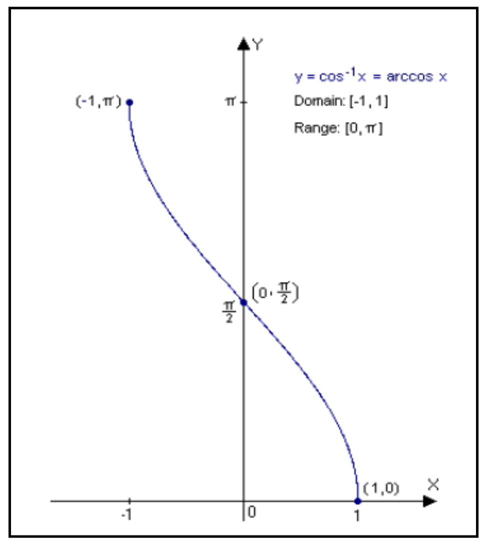


Is the inverse a function now?

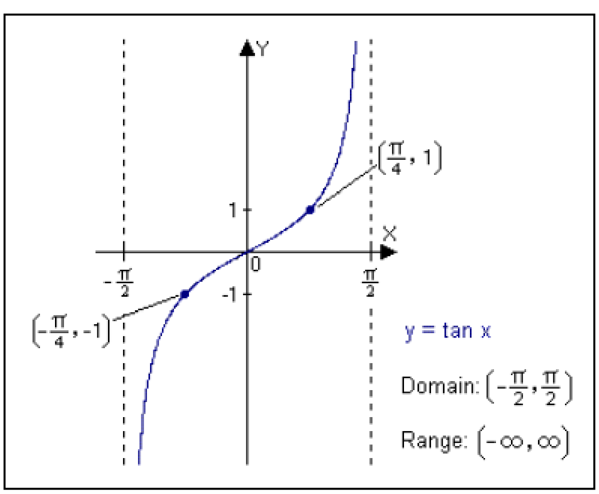
5. Use the following graphs to determine the limited domains on the cosine function used to insure the inverse is a function.

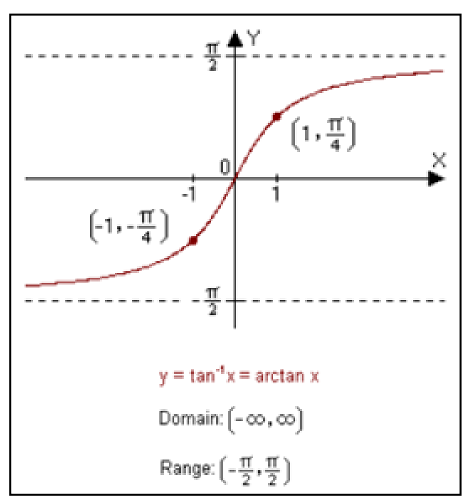
Highlight the axes that represent the angle measure. (on both graphs)





Use the following graphs to determine the limited domains on the tangent function used to insure the inverse is a function.   Mark the axes that represent the angle measure.





We use the names $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ or Arcsin, Arccos, and Arctan to represent the inverse of these functions on the limited domains you explored above. The values in the limited domains of sine, cosine and tangent are called **principal values**. (Similar to the principal values of the square root function.) Calculators give principal values when reporting $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$.

Complete the chart below indicating the domain and range of the given functions.

Function	Domain	Range
$f(\theta) = \sin^{-1}\theta$		
$f(\theta) = \cos^{-1}\theta$		
$f(\theta) = \tan^{-1}\theta$		

The inverse functions do not have ranges that include all 4 domains. Add a column to your chart that indicates the quadrants included in the range of the function. This will be important to remember when you are determining values of the inverse functions.

Function	Domain	Range	Quadrant for Range
$f(\theta) = \sin^{-1}\theta$			
$f(\theta) = \cos^{-1}\theta$			
$f(\theta) = \tan^{-1}\theta$			

Alton felt like he understood inverse trig functions and he quickly evaluated the expressions below. Check Alton's answers to the problems. Mark the problems as correct or incorrect. Correct any problems she missed. (His answers are circled.)

1. $\cos(\tan^{-1} 1) = \frac{3\sqrt{2}}{2}$	2. $\text{Arctan}(-1) = \frac{\pi}{4}$
3. $\sin\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$	4. $\sin(\arctan\sqrt{3} + \arcsin \tfrac{1}{2}) = 2$

Practice

Use what you know about trigonometric functions and their inverses to solve these simple equations. Two examples are included for you. (Unit circles can also be useful.)

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Example 1:

ArcCos $\left(\frac{\sqrt{3}}{2}\right) = x$

The answers will be an angle. Use θ to remind yourself

Let Arccos $\left(\frac{\sqrt{3}}{2}\right) = \theta$

Ask yourself, what angle has a cos value of $\frac{\sqrt{3}}{2}$.

$\left(\frac{\sqrt{3}}{2}\right) = \cos\theta$

Using the definition of Arccos.

$\left(\frac{\pi}{6}\right) = \theta$

Why isn't $\left(\frac{5\pi}{6}\right)$ included?

So, ArcCos $\left(\frac{\sqrt{3}}{2}\right) = \left(\frac{\pi}{6}\right)$

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Example 2:

$\sin(\cos^{-1} 1 + \tan^{-1} 1) = x$

The answer will be a number, not an angle. Simplify parentheses first.

$\theta_1 = \cos^{-1} 1$ $\theta_2 = \tan^{-1} 1$

$\theta_1 = 0$ $\theta_2 = 45°$

$\sin(0 + 45) = x$

Substitution

$\sin(45) = x$

$\frac{\sqrt{2}}{2} = x$

So,

$\sin(\cos^{-1} 1 + \tan^{-1} 1) = \frac{\sqrt{2}}{2}$

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a. $\theta = Cos^{-1}\frac{1}{2}$	b. θ = Arcsin 1
c. $\sin^{-1} 2 = x$	d. $\cos\left(tan^{-1}\sqrt{3} - \sin^{-1}\frac{1}{2}\right) = x$
e. $\cos\left(tan^{-1}\frac{\sqrt{3}}{3}\right) = x$	f. $\sin\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = x$