Remainder Theorem

- If the remainder is 0, then the divisor is a factor of the polynomial.
- Synthetic division can also be used to find the value of a function. This is known as **synthetic substitution**.
- To evaluate a polynomial using synthetic substitution, follow the same process described for synthetic division. For example, given the function $3x^2 - 20x + 12$, if you must determine the value of the function at $x = 3$, use 3 as the a value in the divisor of the synthetic division. The resulting remainder gives the value of the polynomial when evaluated at $x = 3$.
- If a polynomial $p(x)$ is divided by $(x - a)$, then the remainder, r, is equal to $p(a)$.
- This process leads to the **Remainder Theorem**.

Remainder Theorem
For a polynomial $p(x)$ and a number a, dividing $p(x)$ by $x - a$ results in a remainder of $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Example

- Let $p(x) = 3x^2 - 20x + 12$. We can use synthetic substitution (by following the process of synthetic division), to evaluate this function for $x = 3$. The result is the remainder, or –21. Because this result is a number other than 0, the Remainder Theorem allows us to conclude that $(x - 3)$ is not a factor of $p(x)$. Only when the remainder is 0 will $(x - a)$ be a factor of the polynomial. A remainder of any number other than 0 indicates that $(x - a)$ is not a factor of the given polynomial.