

# TRIGONOMETRY II - LESSON TWO

## PART I MULTIPLICATION & DIVISION IDENTITIES

### **Algebraic proofs of trigonometric identities**

In this lesson, we will look at various strategies for proving identities. Try to memorize all the different types, as it will make things much simpler for you when they are mixed together.

### **Type I: Identities with multiplication & division:**

In these proofs, you will need to convert everything to sine and cosine, then use fraction multiplication & division to simplify.

**Example 1:** Prove:  $\sin x \sec x = \tan x$

$$\begin{aligned} \sin x \sec x &= \sin x \left( \frac{1}{\cos x} \right) \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \end{aligned}$$

**Example 2:** Prove:  $\frac{\tan x \cos x}{\sin x} = 1$

$$\begin{aligned} \frac{\tan x \cos x}{\sin x} &= \frac{\left( \frac{\sin x}{\cos x} \right) \cos x}{\sin x} \\ &= \frac{\sin x}{\sin x} \\ &= 1 \end{aligned}$$

**Example 3:** Prove:  $\frac{\csc x}{\cot x} = \sec x$

$$\begin{aligned} \frac{\csc x}{\cot x} &= \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} \\ &= \frac{1}{\sin x} \times \frac{\sin x}{\cos x} \\ &= \frac{1}{\cos x} \\ &= \sec x \end{aligned}$$

## Fraction Review

### **Multiplying Fractions:**

To multiply fractions, simply multiply the numerators together, and the denominators together.

$$\frac{\sin x}{\cos x} \times \frac{1}{\cos x} = \frac{\sin x}{\cos^2 x}$$

### **Canceling:**

When multiplying fractions you will frequently find factors that can be cancelled. You can cancel something on top with something identical on the bottom.

$$\begin{aligned} \frac{\cos x}{\sin x} \times \frac{\sin^2 x}{\cos x} &= \frac{\cancel{\cos x}}{\sin x} \times \frac{\sin^2 x}{\cancel{\cos x}} \\ &= \frac{\sin^2 x}{\sin x} \\ &= \sin x \end{aligned}$$

### **Dividing Fractions:**

When dividing fractions, rewrite the top fraction, then multiply by the reciprocal of the bottom fraction.

$$\frac{\sin x}{\cos x} \div \frac{\sin x}{\cos x} = \frac{\sin x}{\cos x} \times \frac{1}{\cos x} = \frac{\sin x}{\cos^2 x}$$

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For each of the following, write an algebraic proof.

1) Prove:  $\cot x \tan x = 1$

2) Prove:  $\csc x \cos x = \cot x$

3) Prove:  $\frac{\sin x}{\tan x} = \cos x$

4) Prove:  $\frac{1}{\cot x \cos x \tan x} = \sec x$

Identities will always have the following two properties:

1) If you graph the left and right sides, you will obtain exactly the same graph.

2) If you plug in the same angle for  $x$  on both sides, you will obtain exactly the same number.

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5) Prove:  $\frac{\tan x}{\csc x} = \frac{\sin^2 x}{\cos x}$

6) Prove:  $\frac{\tan x}{\sec x} = \sin x$

7) Prove:  $\frac{\cos^2 x}{\cot x} = \sin x \cos x$

8) Prove:  $\frac{\sec x \csc x}{\cot x} = \sec^2 x$

9) Prove:  $\frac{\sec x \csc x}{\csc^2 x} = \tan x$

10) Prove:  $\frac{\tan^2 x \cos x}{2 \sec x} = \frac{1}{2} \sin^2 x$