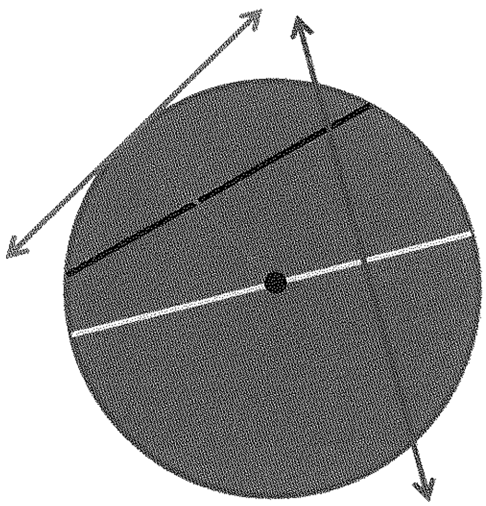
The General Form of a Quadratic Relation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \text{ where } A, B, C \text{ cannot all be zero}$$

The graphs of all conic sections follow this same equation. It should be noted that for our more basic purposes, the B coefficient will always be zero.

Consider…the Circle

Circles should be old news to you, but just as a quick review let's see if you can remember their important parts.



For the circle to the left, label the following features on the diagram:

Center
Diameter
Radius
Chord
Secant Line
Tangent Line
Point of Tangency

Now let's try some very quick circle review using a few of these terms.

(a) What relationship do the points making up the graph of a circle have to the center?

(b) What relationship do the radius and the diameter of a circle have?

(c) What is a tangent line and what relationship does it have to the radius that it meets at the point of tangency?

(d) If a circle has a diameter with endpoints $(-2, -5)$ and $(3, 4)$, what is...

(i) the diameter of the circle?

(ii) the center of the circle?

(iii) the radius of the circle?

(iv) the slope of the radius from the center to $(3, 4)$?

(v) the equation of the tangent line that intersects the circle at the point $(-2, -5)$?

If you answered (a) correctly, you know that the locus of points making up a circle are equidistant from the circle's center. This leads to an important idea about the center – it serves as the focus of the circle. The points making up the circle are all entirely dependent upon the location of that important focal point.

Now let's review the standard form of the equation describing a circle.

Standard Form of a Circle:

$$(x-h)^2+(y-k)^2=r^2 \text{ with center at } (h,k) \text{ and radius } r$$

This is the most useful form of a circle in terms of recognizing important pieces and for graphing and was the emphasis of your previous work with circles.

Let's try writing a few equations in standard form.

1. Write the equation for the circle with a diameter containing the endpoints $(-3,0)$ and $(3,0)$.

2. Write the equation for the entire set of points that are 4 units away from $(1,-5)$.

3. Write the equation of the circle with a radius from the center at $(2,7)$ to an endpoint at $(6,5)$.

And now let's review how to take a circle in a different form and change it to the more useable standard form. For example, let's look at the following:

$$x^2+y^2+6x-2y+1=0$$

Notice that this circle is presented in the **general form** $Ax^2+Cy^2+Dx+Ey+F=0$ where $A=1, C=1, D=6, E=-2,$ and $F=1$. As you work through the next set of problems, see if you recognize any patterns in the coefficients for general form, and then see if you can find other patterns using the general form equations for other conic sections. In any case, this general form is not useful in terms of graphing, or picking out the radius, diameter, or center. So we need to put the equation into standard form. To do this by completing the square, first group like variables together and move the constant to the other side of the equation.

$$x^2+y^2+6x-2y+1=0$$

$$x^2 + 6x + y^2 - 2y = -1$$

Once we've gotten like variables together and sent the constant to the other side, we have to complete the square by taking the coefficient of the linear term for both variables, dividing it by 2, and squaring the quotients. Add both of these squares to both sides of your equation.

$$\left(x^2 + 6x + \left(\frac{6}{2}\right)^2\right) + \left(y^2 - 2y + \left(\frac{-2}{2}\right)^2\right) = -1 + \left(\frac{6}{2}\right)^2 + \left(\frac{-2}{2}\right)^2$$

$$(x^2 + 6x + 9) + (y^2 - 2y + 1) = 9$$

Now, all we must do is factor our two perfect square trinomials and we'll have standard form.

$$(x + 3)^2 + (y - 1)^2 = 9$$

Now we know that the circle has a center of $(-3, 1)$ and a radius of 3, facts not obvious from the original general form.

Put the following equations into standard form.

1. $x^2 + y^2 - 4x + 12y - 6 = 0$

2. $x^2 - 6x = y - y^2 + 7$

3. $\frac{7x^2}{3} + \frac{7y^2}{3} = 1$