Graphs of Polynomial Notes

Turning Points

- A **turning point** of a function is a point where the graph of the function changes from sloping upward to sloping downward or, alternatively, from sloping downward to sloping upward.
- To determine the maximum number of turning points of a function, subtract 1 from the highest degree of the polynomial. In other words, find $n - 1$.
- For instance, the polynomial function $y = 3x^7 + 9x^3 - x + 4$ can have no more than $7 - 1$, or 6, turning points.
- The maximum number of turning points does not necessarily indicate the actual number of turning points of a function, just that it can have no more than that number. Some functions may have fewer turning points than the number calculated.
- A turning point corresponds to a **local maximum**, the greatest value of a function for a particular interval of the function, or a **local minimum**, the least value of a function for a particular interval of the function. A local maximum may also be referred to as a **relative maximum** and a local minimum may also be referred to as a **relative minimum**.

Roots of a Polynomial Function

- The highest degree of the polynomial determines the maximum number of **roots**, or x-intercepts of a function.
- A polynomial function with a degree of 10 could have up to 10 roots, but could also have 0 to 9 roots, depending on the specific equation.
- Recall that real numbers include all rational and irrational numbers, but do not include imaginary and complex numbers.

Sketching a Polynomial Function

- Being able to identify the general end behavior, the possible number of turning points, and the maximum number of roots of a polynomial function can be helpful in creating a rough sketch of the function.
- Start by choosing at least six x-values that are both positive and negative. It is also useful to choose the value of 0.
- As you've done in previous courses, substitute each chosen x-value into the given function and evaluate to determine the corresponding y-value. Then, plot the points on a graph.
- Be sure to smoothly connect all chosen points to illustrate the graph of the function.
- Graphing calculators are especially helpful when sketching a complicated function.

Review

Name: Date:

UNIT 2 • POLYNOMIAL FUNCTIONS
Lesson 1: Polynomial Structures and Operating with Polynomials

Assessment

Pre-Assessment

Circle the letter of the best answer.

1. What is the degree of the polynomial $5x^6-2x^4+x^3+10x$?
   a. 1
   b. 5
   c. 6
   d. 10

2. What is the result of $(4x^5-2x+8)+(-7x^2+9x+3)$?
   a. $11x^2+11x+11$
   b. $3x^3+7x+11$
   c. $-3x^5+7x+11$
   d. $4x^5-7x^2+7x+11$

3. What is the result of $(-x^6+10x^3+4x^2+22)-(5x^6-2x^2+16)$?
   a. $-6x^6+10x^3+6x^2+6$
   b. $4x^6+10x^3+2x^2+6$
   c. $5x^6+12x^3+4x^2+6$
   d. $4x^6+10x^3+2x^6+38$

4. What is the result of $(x^3+6x)(2x^2+1)$?
   a. $2x^5+x^3$
   b. $2x^5+13x^3+6x$
   c. $12x^3+6x$
   d. x^3+2x^2+6x+1

5. What is the result of $(-3x^4+x^2-1)(5x^2+4)$?
   a. $-15x^6-12x^4$
   b. $-15x^6-7x^4-x^2-4$
   c. $-15x^6+17x^4+9x^2+4$
   d. $2x^4-5x^2-4$

Name: Date:

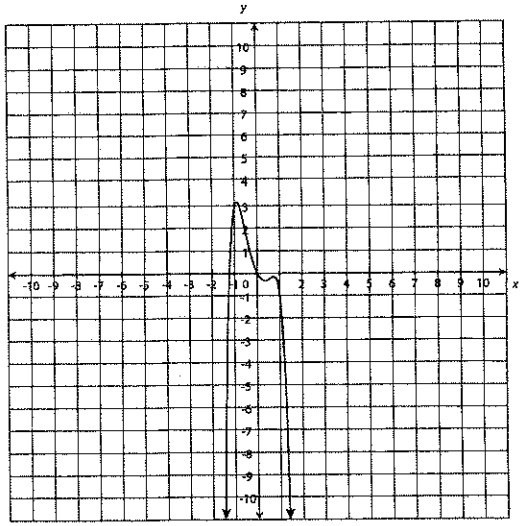
UNIT 2 • POLYNOMIAL FUNCTIONS
Lesson 3: Graphing Polynomial Functions

Assessment

Pre-Assessment

Circle the letter of the best answer.

6. What must be true about the degree and leading coefficient of the graphed polynomial?



   a. The polynomial is an odd-degree polynomial and has a positive coefficient.
   b. The polynomial is an odd-degree polynomial and has a negative coefficient.
   c. The polynomial is an even-degree polynomial and has a positive coefficient.
   d. The polynomial is an even-degree polynomial and has a negative coefficient.

continued

7. What is the remainder when $(x^3 - 3x^2 + 6x - 5)$ is divided by $(x - 2)$?

a. -37

b. 3

c. -7

d. 5

8. Which polynomial function has zeros of -3, 1, and 2?

a. $f(x) = x^3 - 7x + 6$

b. $f(x) = x^3 - 7x - 6$

c. $f(x) = x^3 + 6x^2 + 11x + 6$

d. $f(x) = x^3 - 6x^2 + 11x - 6$

9. One root of the equation $x^3 - 5x^2 + 11x - 15$ is 3. What are the other two roots?

a. $2 + i$ and $2 - i$

b. $-1 + 2i$ and $-1 - 2i$

c. $1 + 2i$ and $1 - 2i$

d. $-2 + i$ and $-2 - i$

10. What are all the possible rational roots for the polynomial equation $4x^3 + 2x^3 + 5x + 8 = 0$?

a. $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

b. $\pm\frac{1}{8}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 2, \pm 4$

c. $\pm\frac{1}{4}, \pm 1, \pm 2, \pm 8$

d. $\pm\frac{1}{8}, \pm\frac{1}{2}, \pm 1, \pm 8$

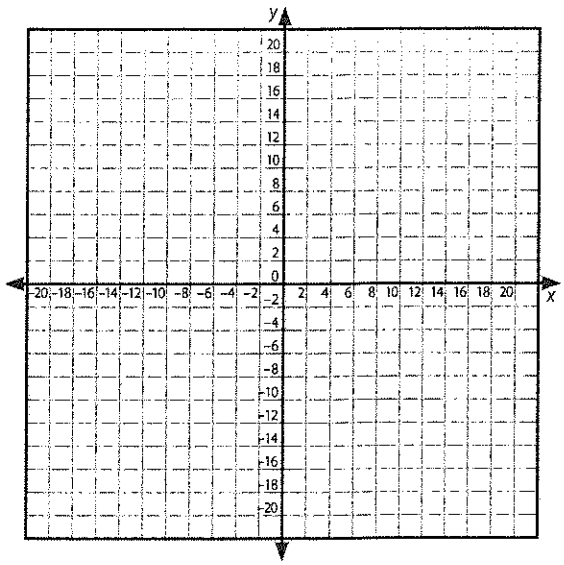
Station 4

Work with your group to answer the questions about each polynomial function. Then use a graphing calculator to find the graph of the function. Sketch the graphs.

1. $f(x) = x^3 + 3x^2 - 4x - 12$

   a. ~~Factor the polynomial to~~ find the zeros of the function.

   b.

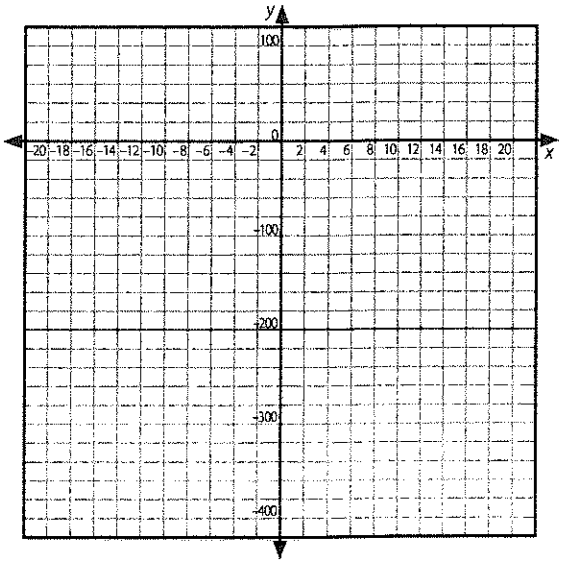


Degree
Domain
Range
End Behavior
turning points
Symmetry

2. $f(x) = 3x^4 + 5x^3 - 49x^2 + 11x + 30$

   a. ~~Factor the polynomial to~~ find the zeros of the function.

   b.



Degree
Domain
Range
End Behavior
Turning Points
Symmetry

continued