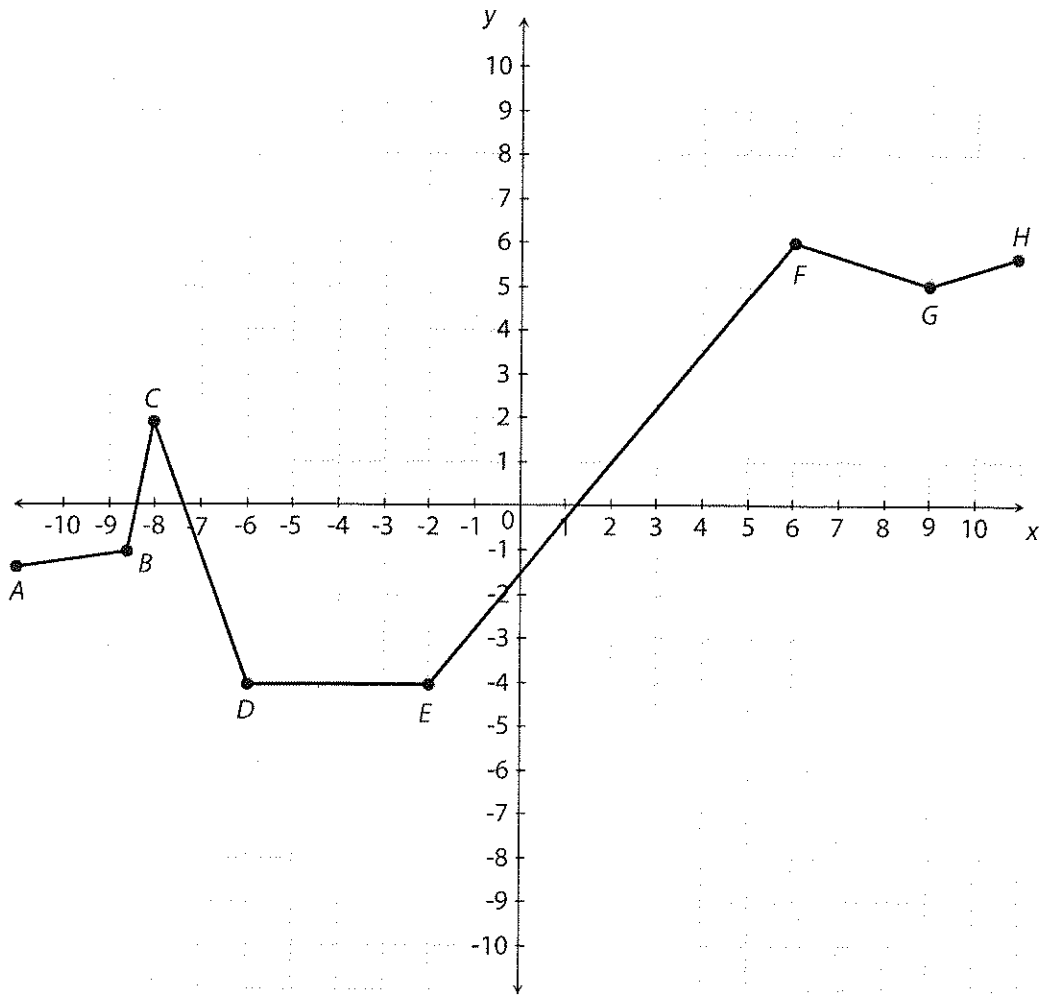
Lesson 1.9.3: Proving the Midsegment of a Triangle

Warm-Up 1.9.3

A portion of the Los Angeles marathon course is mapped on the coordinate plane shown. Each unit represents 600 feet. The locations of major landmarks along the course are labeled by points.



1. Officials need to verify the distance between points C and D. What is this distance?

2. A water station is planned midway between points E and F. What is the location of the water station?

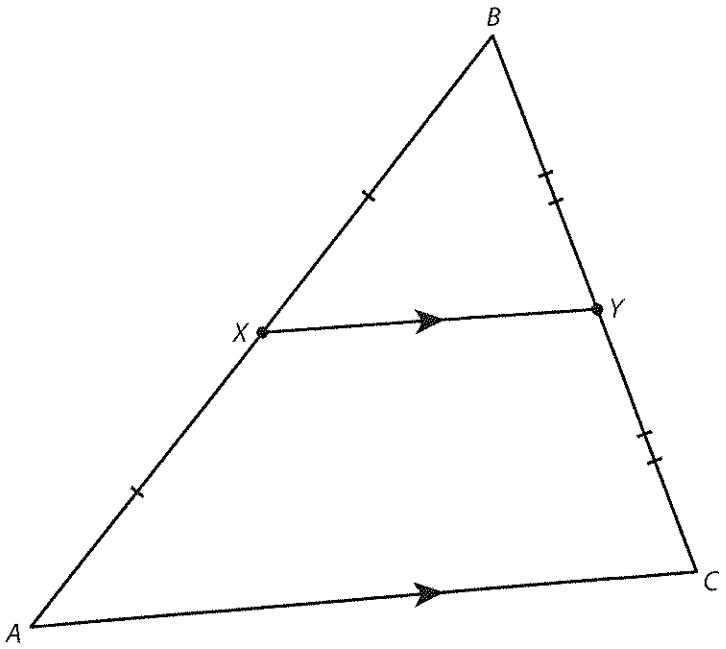
Instruction

- The midsegment of a triangle is parallel to the third side of the triangle and is half as long as the third side. This is known as the Triangle Midsegment Theorem.

Theorem

Triangle Midsegment Theorem

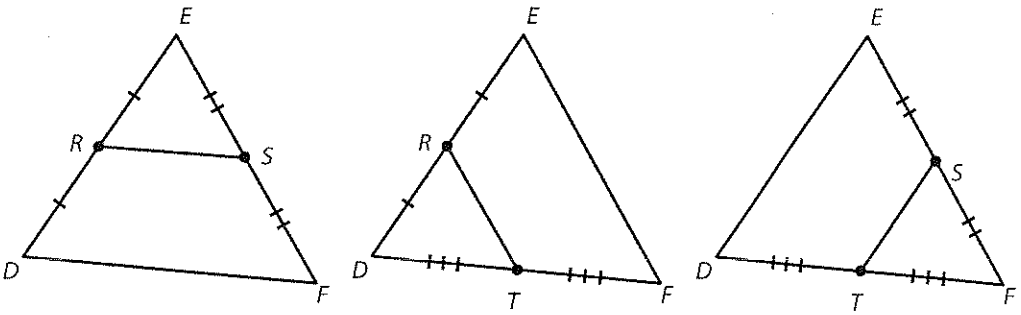
A midsegment of a triangle is parallel to the third side and is half as long.



$\overline{AC} \parallel \overline{XY}$

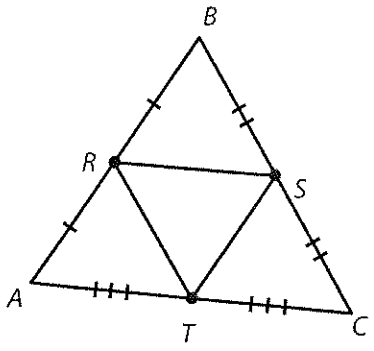
$XY = \frac{1}{2}AC$

- Every triangle has three midsegments.



Instruction

- When all three of the midsegments of a triangle are connected, a **midsegment triangle** is created.



- In the diagram above, $\triangle ABC \sim \triangle TSR$.
- **Coordinate proofs**, proofs that involve calculations and make reference to the coordinate plane, are often used to prove many theorems.

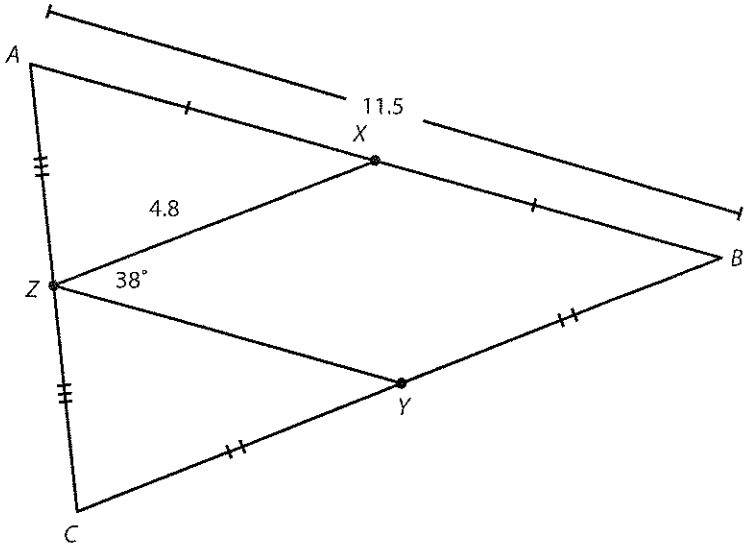
Common Errors/Misconceptions

- assuming a segment that is parallel to the third side of a triangle is a midsegment
- incorrectly writing and solving equations to determine lengths
- incorrectly calculating slope
- incorrectly applying the Triangle Midsegment Theorem to solve problems
- misidentifying or leaving out theorems, postulates, or definitions when writing proofs

Lesson 1.9.3 Midsegment Theorem
Examples

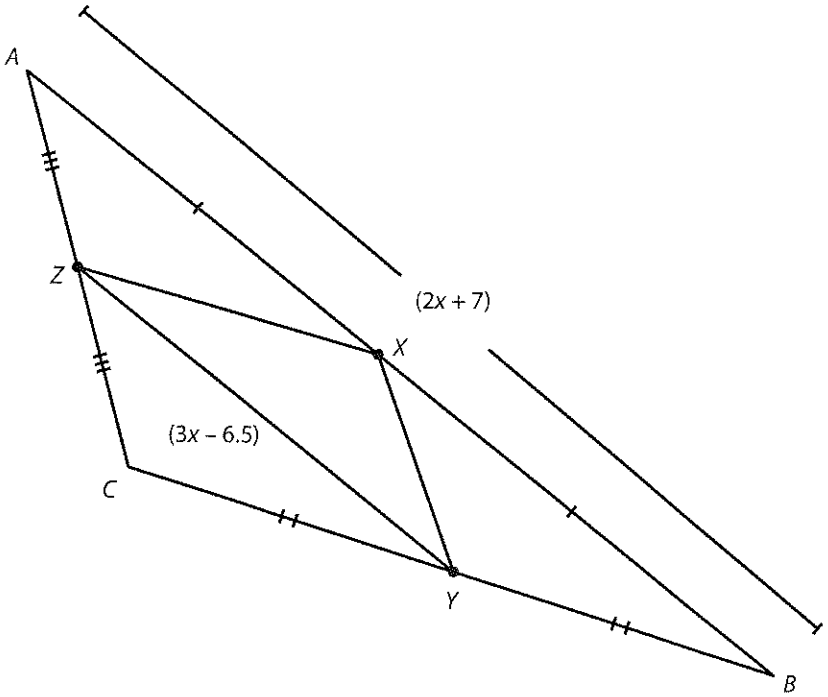
Example 1

Find the lengths of BC and YZ and the measure of $\angle AXZ$.



Example 2

If $AB = 2x + 7$ and $YZ = 3x - 6.5$, what is the length of AB?



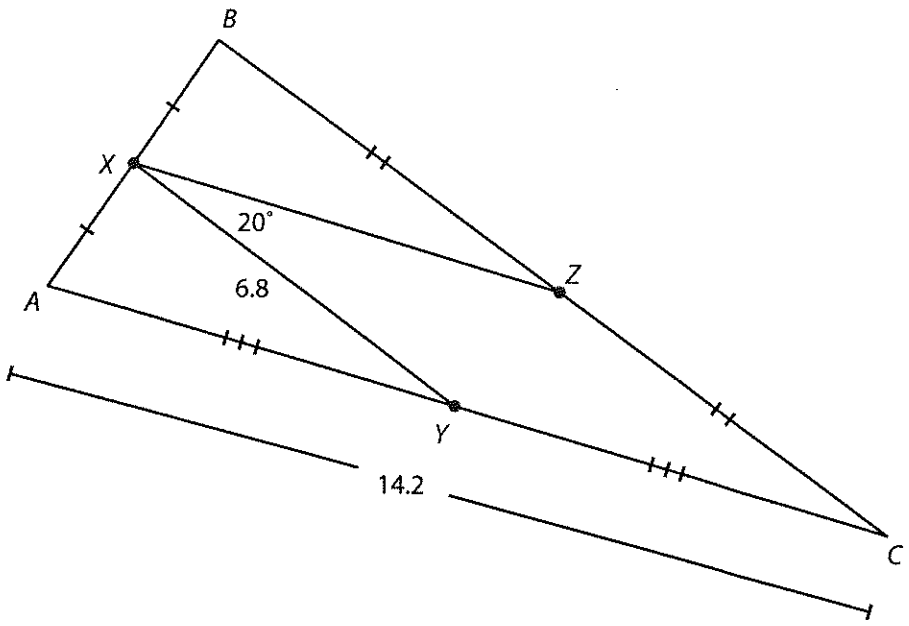
Example 3

The midpoints of a triangle are $X(-2, 5)$, $Y(3, 1)$, and $Z(4, 8)$. Find the coordinates of the vertices of the triangle.

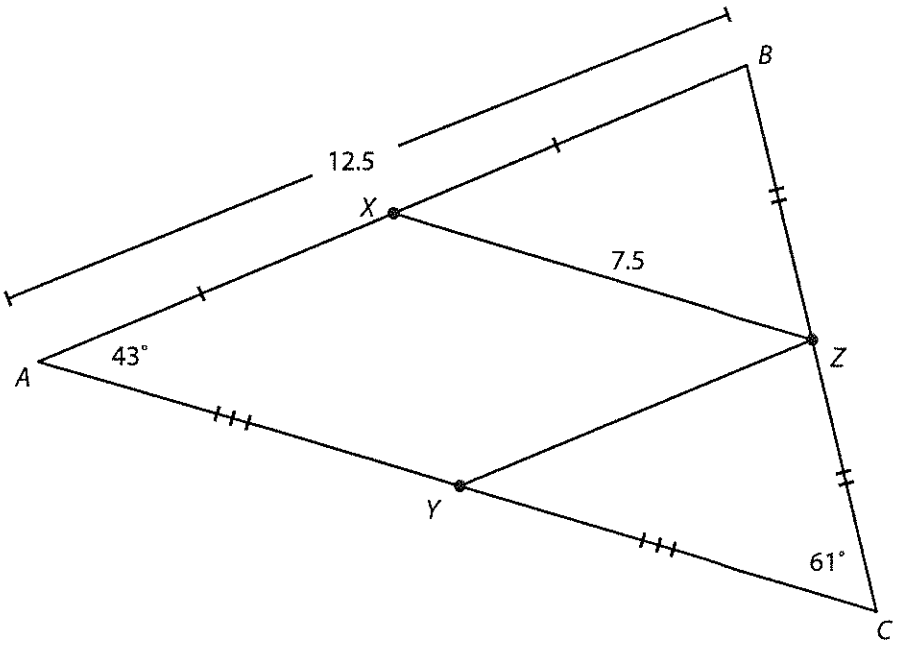
Practice 1.9.3: Proving the Midsegment of a Triangle

Use your knowledge of midsegments to solve each problem.

1. Find the lengths of BC and XZ and the measure of $\angle BZX$.



2. Find the lengths of AC and YZ and the measure of $\angle XZY$.



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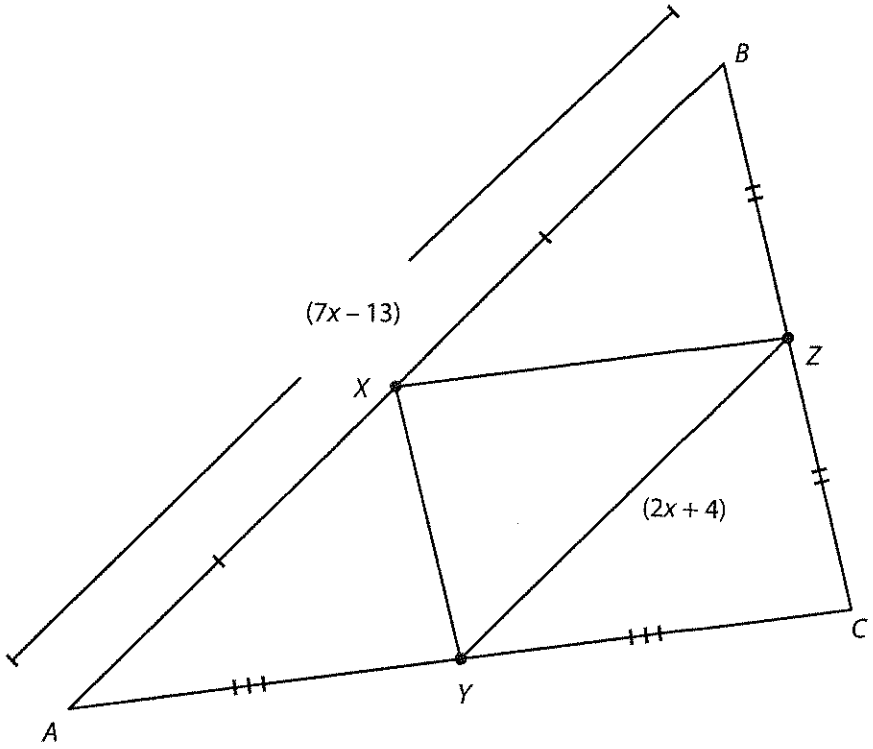
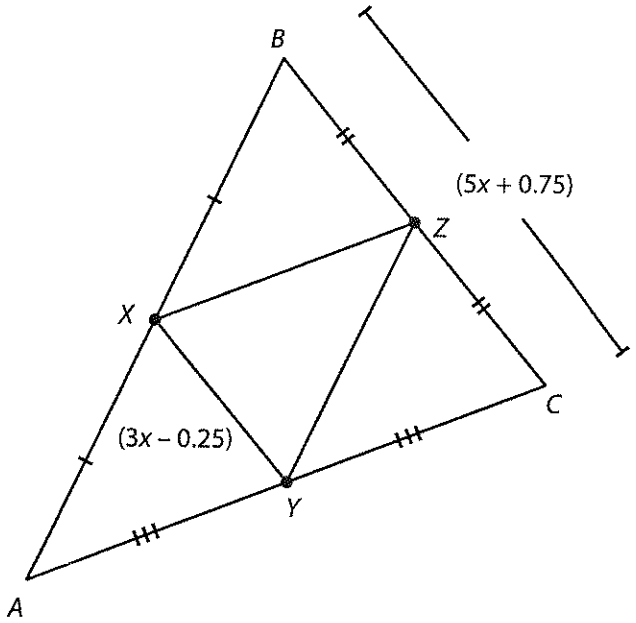
3. If $AB = 7x - 13$ and $YZ = 2x + 4$, what is the length of YZ?



4. If $BC = 5x + 0.75$ and $XY = 3x - 0.25$, what is the length of BC?



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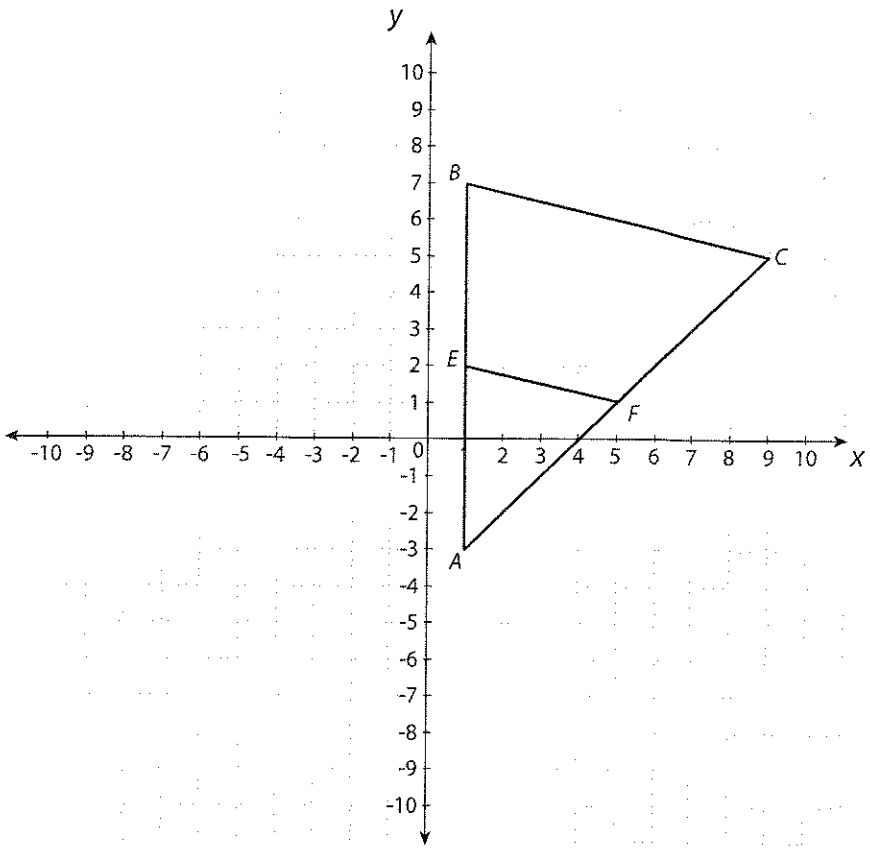
5. The midpoints of a triangle are $X(-5, -4)$, $Y(-3, 2)$, and $Z(5, 2)$. Find the coordinates of the vertices of the triangle.

6. The vertices of a triangle are $A(-5, -4)$, $B(1, 10)$, and $C(9, 0)$. Find the coordinates of the midpoints of the triangle.

7. Use a coordinate proof to show that $\overline{EF} \parallel \overline{BC}$ and $EF = \frac{1}{2}BC$.



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