

PROVING THE LAW OF COSINES

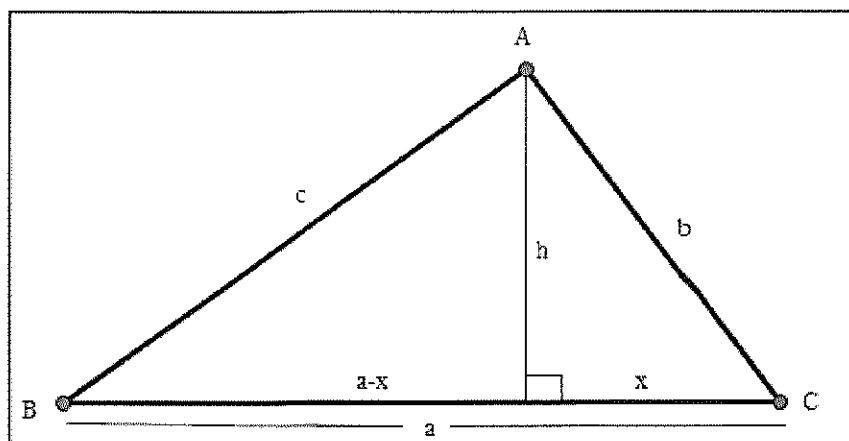
During a baseball game an outfielder caught a ball hit to dead center field, 400 feet from home plate. If the distance from home plate to first base is 90 feet, how far does the outfielder have to throw the ball to get it to first base?



1. Model the problem with a picture. Be sure to label information that you know.

2. Do you have enough information to solve the problem? If not, what is missing?

Typically, you have solved triangles that are right triangles. This is a case where we do not have a right triangle to solve. We know two sides and one included angle. In this task, you will develop a method for solving triangles like this using trigonometry. We will come back to the baseball example later. For now, consider the triangle below. Follow these steps to derive a way to solve for c knowing just that much information. For this example, assume we know measurements for segments a , b , and angle C .



3. What does segment h represent? What are its properties and what does it do to the large triangle?

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4. Write an equation that represents c^2 . Explain the method you used.

5. Now write an equation that represents h^2 in terms of b and x . Substitute this expression into the expression you wrote in #4. Expand and simplify.

6. Now write an expression that represents x in terms of the angle C . Substitute this expression into the equation you wrote in #5. Simplify completely.

Your answer to #6 is one of three formulas that make up the **Law of Cosines**. Each of the formulas can be derived in the same way you derived this one by working with each vertex and the other heights of the triangle.

Law of Cosines

Let a , b , and c be the lengths of the legs of a triangle opposite angles A , B , and C . Then,

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

$$b^2 = a^2 + c^2 - 2ac(\cos B)$$

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

These formulas can be used to solve for unknown lengths and angles in a triangle.

7. Solve the baseball problem at the beginning of this task using the Law of Cosines.

