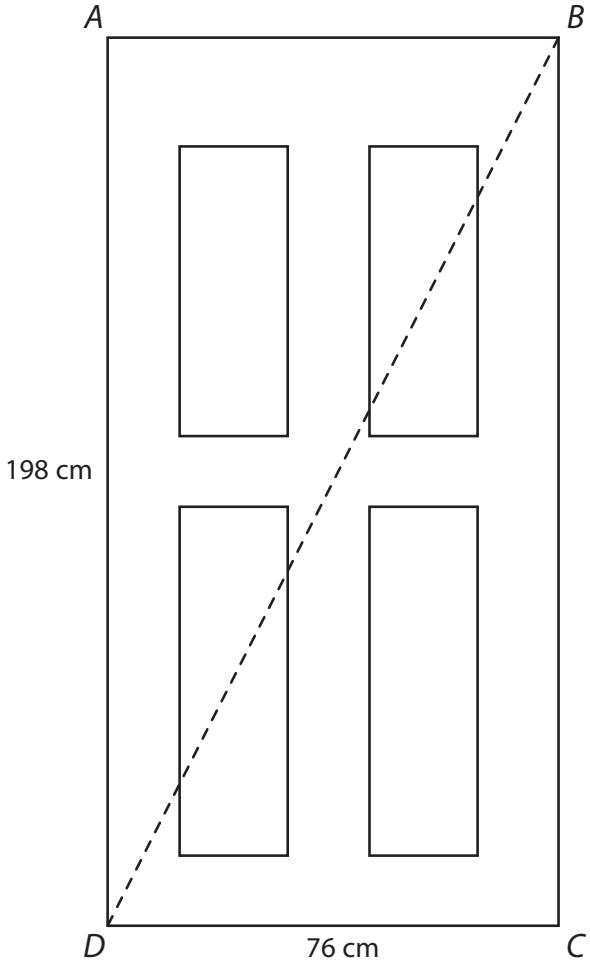
Lesson 1.7.3: Proving the Pythagorean Theorem Using Similarity

Warm-Up 1.7.3

Woodworkers must accurately cut and assemble each piece of wood to ensure that a project is "square." Every vertical piece should intersect every horizontal piece at a 90° angle. To determine if a project is square, woodworkers use the Pythagorean Theorem, which states that the sum of the squares of the two legs of a right triangle is equal to the square of the longest side. If the lengths of the diagonals are equal, then the project is square. Use the diagram below of a door to solve the problems that follow.



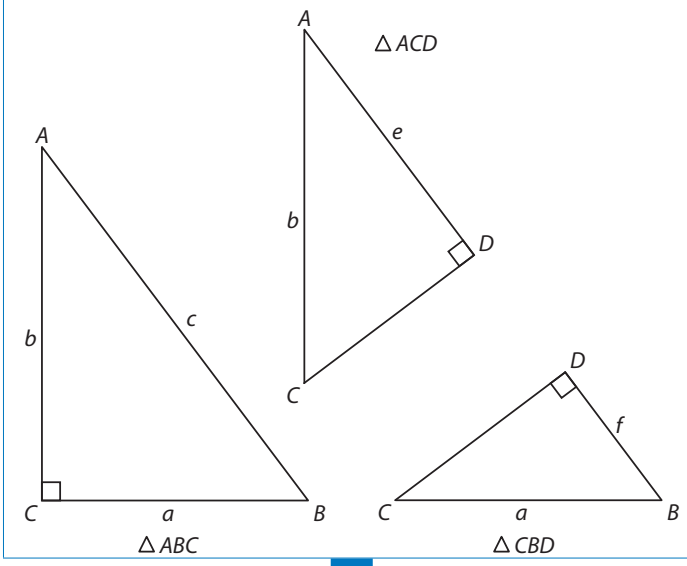
1. A woodworker measured the length of one diagonal of the wooden door, $\overline{BD}$, to be 212 cm. The woodworker measured the length of $\overline{AD}$ to be 198 cm and the length of $\overline{DC}$ to be 76 cm. Calculate the length of $\overline{AC}$.
2. Is $\overline{BD}$ congruent to $\overline{AC}$?
3. Is the door "square"? Explain your answer.

Types of Proofs

- **Paragraph proofs** are statements written out in complete sentences in a logical order to show an argument.
- **Flow proofs** are a graphical method of presenting the logical steps used to show an argument.
- In a flow proof, the logical statements are written in boxes and the reason for each statement is written below the box.
- Another accepted form of proof is a **two-column proof**.
- Two-column proofs include numbered statements and corresponding reasons that show the argument in a logical order.
- Two-column proofs appear in the Guided Practice examples that follow.

2. Identify the similar triangles.

It is often helpful to redraw the triangles.

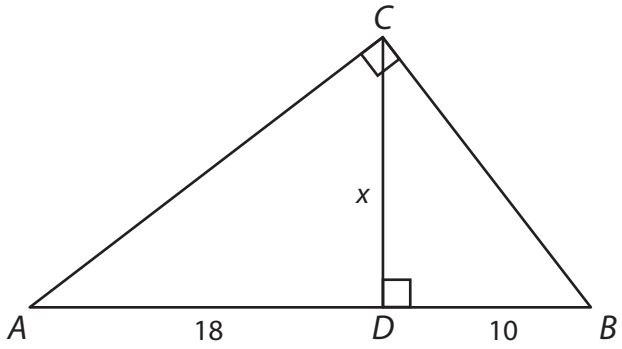


3. Create the two-column proof.

Statements	Reasons
1. $\triangle ABC$ with right $\angle C$	1. Given
2. $\triangle ABC \sim \triangle ACD$ $\triangle ABC \sim \triangle CBD$	2. If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and each other.
3. $\frac{c}{a}=\frac{a}{f}; \frac{c}{b}=\frac{b}{e}$	3. Definition of similar triangles; corresponding sides are proportional.
4. $cf = a^2; ce = b^2$	4. Multiplication Property of Equality
5. $cf + ce = a^2 + b^2$	5. Addition Property of Equality
6. $c(f + e) = a^2 + b^2$	6. Distributive Property of Equality
7. $e + f = c$	7. Segment Addition Postulate
8. $c(c) = a^2 + b^2$ or $c^2 = a^2 + b^2$	8. Substitution Property

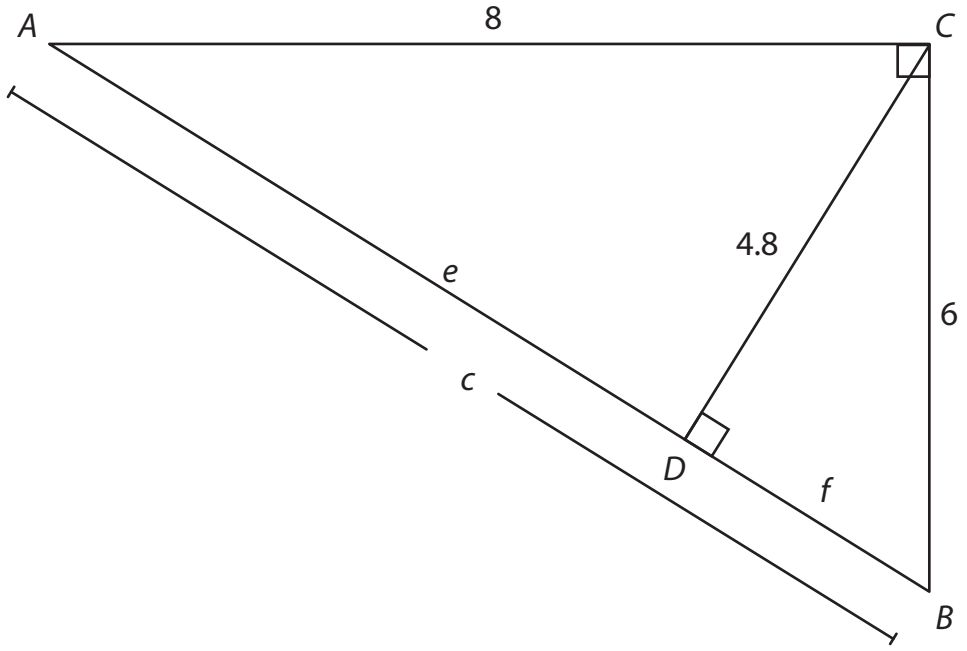
Example 2

Find the length of the altitude, x, of $\triangle ABC$.



Example 3

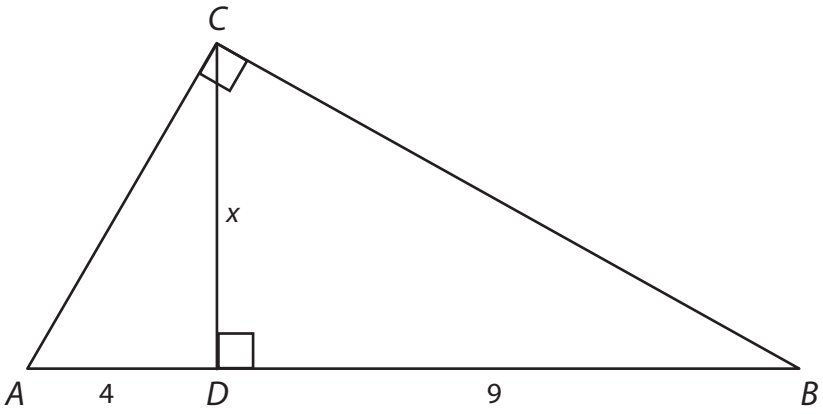
Find the unknown values in the figure.



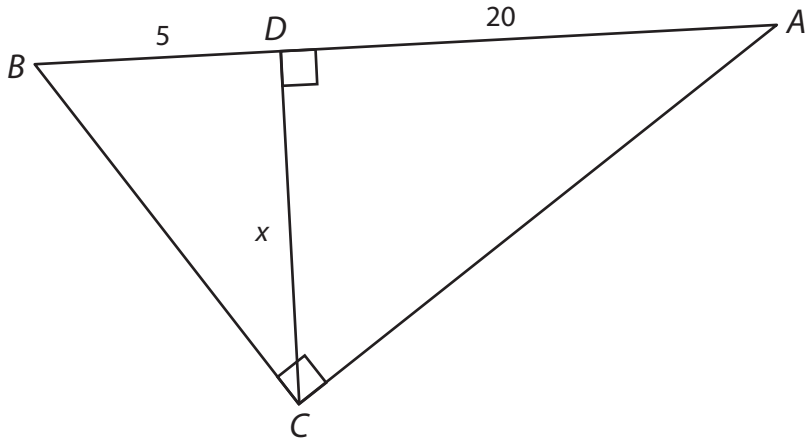
Practice 1.7.3: Proving the Pythagorean Theorem Using Similarity

Find the unknown length(s) in each figure.

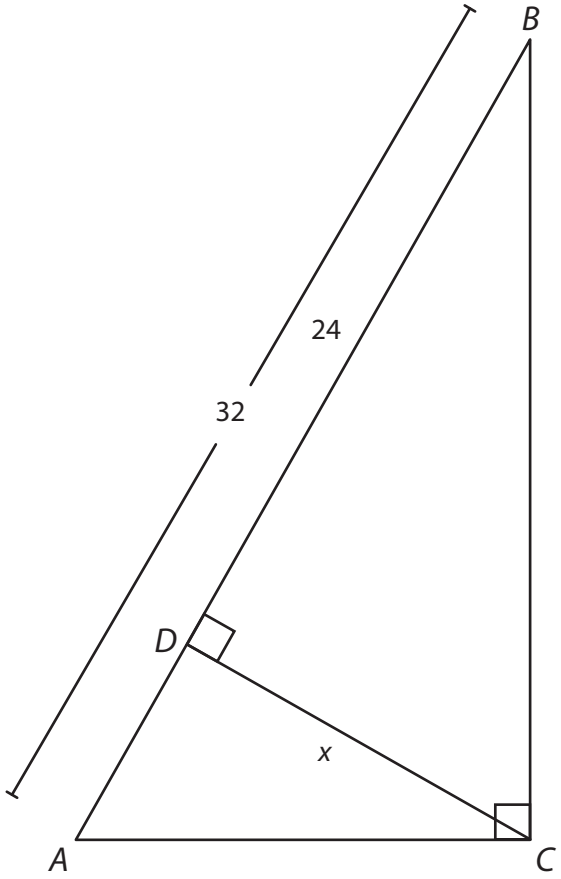
1.



2.



3.



4.