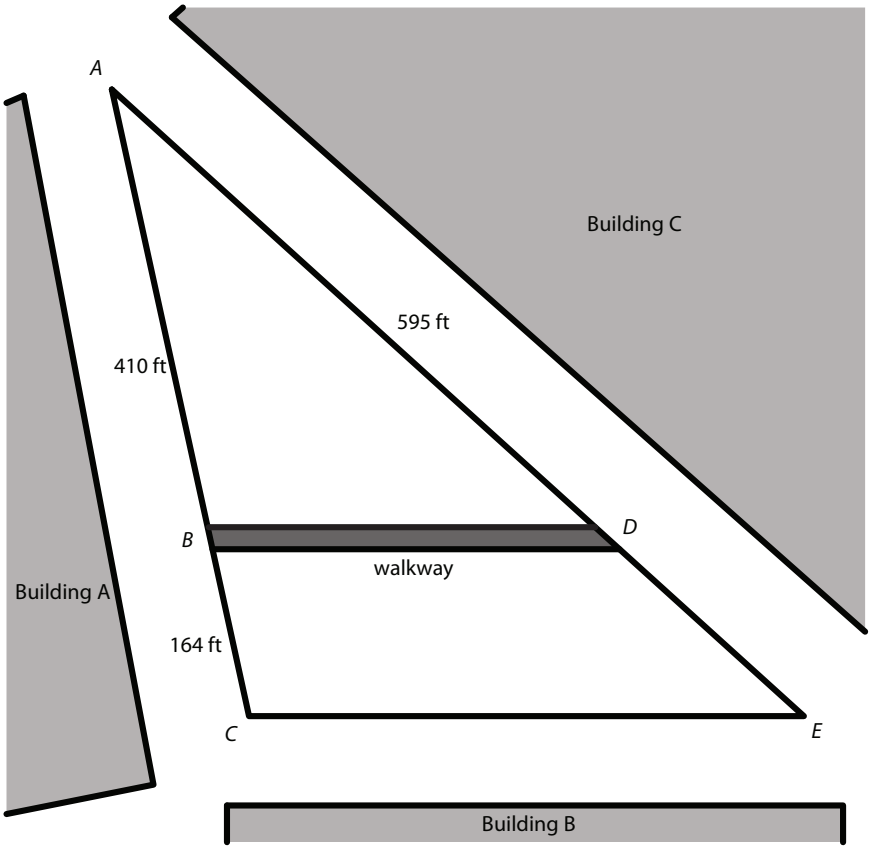
Lesson 1.7.4: Solving Problems Using Similarity and Congruence

Warm-Up 1.7.4

Three buildings border a triangular courtyard as shown in the diagram. A walkway runs parallel to the edge of the courtyard labeled $\overline{CE}$. Landscapers would like to install a picket fence along the outside of the courtyard with the exception of the walkway. The fencing comes in 8-foot lengths.



1. Identify the similar triangles.

2. While preparing the sketch of the courtyard, landscapers forgot to measure the length of the courtyard represented by $\overline{DE}$. What is the length of $\overline{DE}$?

3. How many sections of fencing are needed?

Key Concepts

Similarity

- Similarity statements include Angle-Angle (AA), Side-Angle-Side (SAS), and Side-Side-Side (SSS).
- These statements allow us to prove triangles are similar.
- Similar triangles have corresponding sides that are proportional.
- It is important to note that while both similarity and congruence statements include an SSS and an SAS statement, the statements do not mean the same thing.
- Similar triangles have corresponding sides that are proportional, whereas congruent triangles have corresponding sides that are of the same length.

Triangle Theorems

- The Triangle Proportionality Theorem states that if a line parallel to one side of a triangle intersects the other two sides of the triangle, then the parallel line divides these two sides proportionally.
- This theorem can be used to find the lengths of various sides or portions of sides of a triangle.
- It is also true that if a line divides two sides of a triangle proportionally, then the line is parallel to the third side.

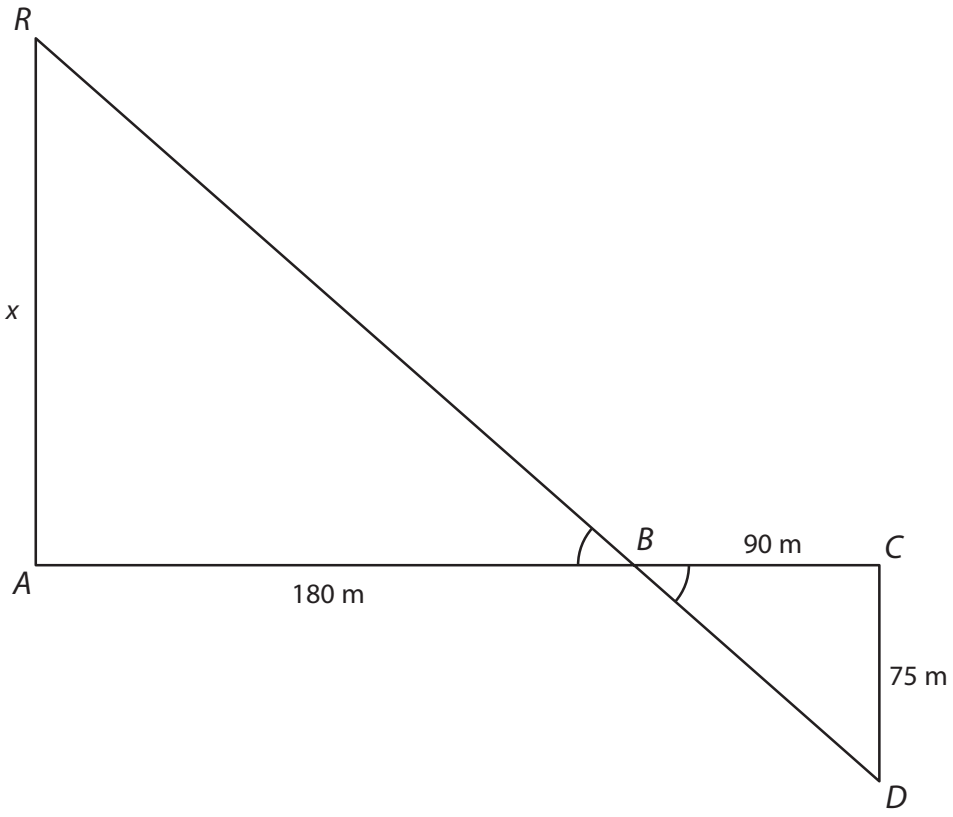
- The Triangle Angle Bisector Theorem states if one angle of a triangle is bisected, or cut in half, then the angle bisector of the triangle divides the opposite side of the triangle into two segments that are proportional to the other two sides of the triangle.
- The Pythagorean Theorem, written symbolically as $a^2 + b^2 = c^2$, is often used to find the lengths of the sides of a right triangle, which is a triangle that includes one 90˚ angle.
- Drawing the altitude, the segment from the right angle perpendicular to the line containing the opposite side, creates two smaller right triangles that are similar.

Example 1

A meterstick casts a shadow 65 centimeters long. At the same time, a tree casts a shadow 2.6 meters long. How tall is the tree?

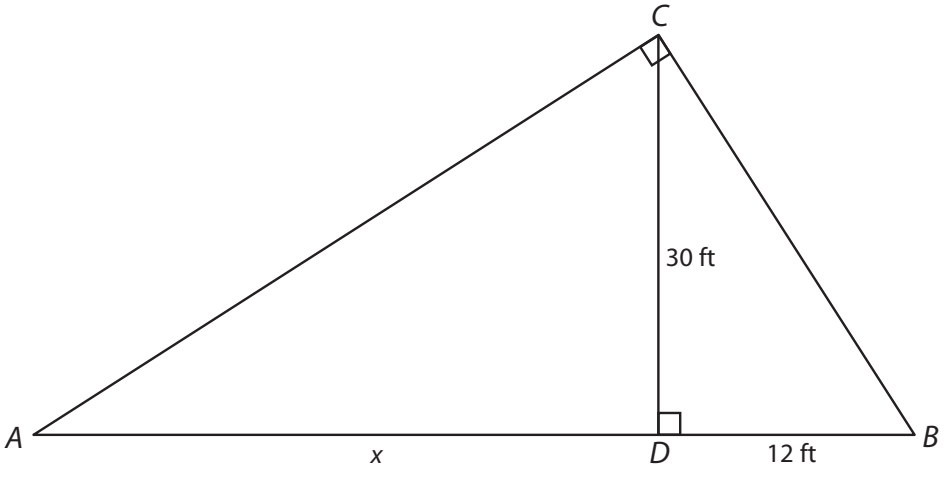
Example 2

Finding the distance across a canyon can often be difficult. A drawing of similar triangles can be used to make this task easier. Use the diagram to determine $\overline{AR}$, the distance across the canyon.



Example 3

To find the distance across a pond, Rita climbs a 30-foot observation tower on the shore of the pond and locates points A and B so that $\overline{AC}$ is perpendicular to $\overline{CB}$. She then finds the measure of $\overline{DB}$ to be 12 feet. What is the measure of $\overline{AD}$, the distance across the pond?



Problem-Based Task 1.7.4: Too Tall?

Parks directors routinely assess the health of the trees in recreation areas. If trees are found to be diseased, they are often treated. If trees become too weak, they are removed before they become a danger to people and structures. Gorge Park is a rectangular park measuring 400 feet by 200 feet and is enclosed by a fence. A diseased tree needing removal stands in the center of the park. Tree removers must avoid having the tree fall on the fence. If necessary, the tree can be trimmed prior to being cut down. The 6-foot-tall parks director measured the length of the shadow cast by the tree to be 147 feet and the length of his own shadow to be 9 feet. Does the tree's trunk need to be trimmed prior to cutting it down to avoid damaging the fence?

Practice 1.7.4: Solving Problems Using Similarity and Congruence

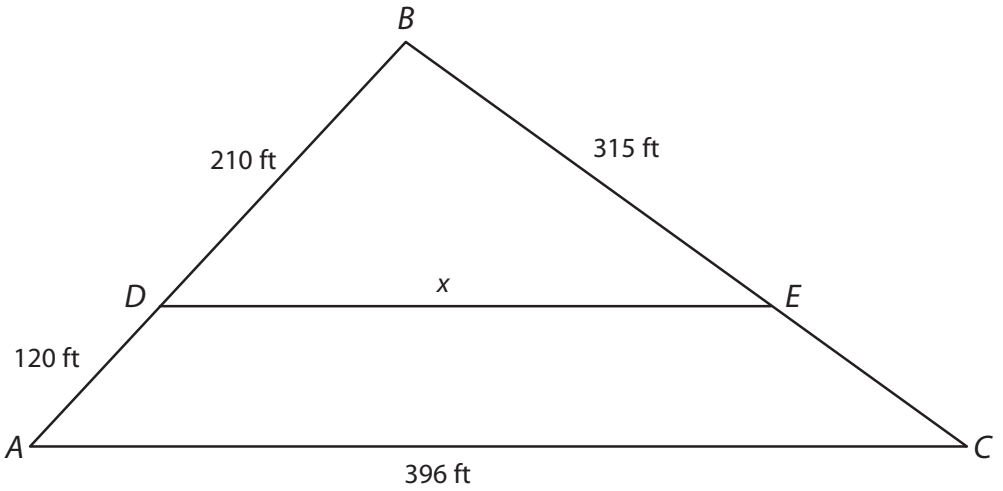
Use what you have learned about similar triangles to solve each problem.

1. A flat-roofed garage casts a shadow that is 9 meters long. At the same time, a 1.8-meter lamppost casts a shadow that is 2.7 meters long. What is the height of the garage?

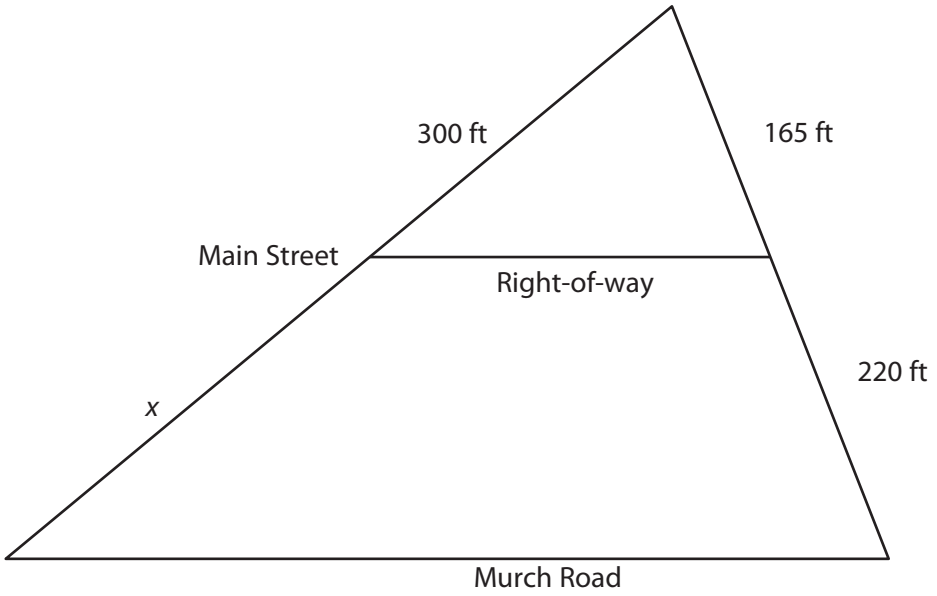
2. A 12-foot statue casts a shadow that is 5 feet long. At the same time, a fence post casts a shadow that is 1.25 feet long. What is the height of the fence post?

For problems 3–10, use the information and the diagrams to solve each problem.

3. A piece of decorative trim is added to an asymmetrical roofline. What is the length of the decorative trim, $\overline{DE}$?



4. A right-of-way parallel to Murch Road is to be constructed on a triangular plot of land. What i the length of the plot of land along Main Street between Murch Road and the right-of-way?



5. To measure $\overline{BC}$, the distance across a lake, a surveyor stands at point *A* and locates points *B*, *C*, *D*, and *E*. What is the distance across the lake?