Lesson 1 Circles and their Relationships among Central Angles, Arcs, and Chords

In this unit you will study properties of circles. We will be keeping a "**Circle Book**" or list that includes the definitions and theorems addressed in each task. With each definition and theorem you enter, you should also include an illustrative sketch.

We will begin by re-visiting the definition of a circle.

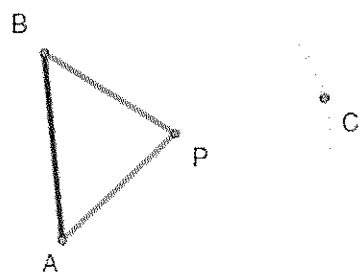
1. Use a compass to construct a circle on an unlined sheet of paper.
   Label the center of your circle with a Capital Letter.

a. What information do you need to determine a unique circle?

b. Prove that all circles are similar.

c. Use your answer to *item a* to help you write a definition of a circle.

Now we will introduce some notation and terminology needed to study circles.
Consider the figure at right.
$m\angle APB = 75°$



Circles are identified by the notation $\odot P$, where *P* represents the point that is the center of the circle.

A **central angle** of a circle is an angle whose vertex is at the center of the circle.
$\angle APB$ is a central angle of $\odot P$.

A portion of a circle's circumference is called an **arc**. An arc is defined by two endpoints and the points on the circle between those two endpoints. If a circle is divided into two unequal arcs, the shorter arc is called the **minor arc** and the longer arc is called the **major arc**. If a circle is divided into two equal arcs, each arc is called a **semicircle**.

In our figure, we call the portion of the circle between and including points *A* and *B*, arc *AB* notated by $\overset{\frown}{AB}$. We call the remaining portion of the circle arc *ACB, or* $\overset{\frown}{ACB}$. Note that major arcs are usually named using three letters.

We say that the central angle $\angle APB$ *intercepts* or has $\overset{\frown}{AB}$. We also say that $\overset{\frown}{AB}$ *subtends* or has the central angle $\angle APB$. Note that when we refer to the arc of a central angle, we usually mean the minor arc unless otherwise stated.

Arcs are measured in two different ways - using degree measure and using linear measure. Usually when we refer to the **measure** of an arc, we are referring to the degree measure. The **measure** of a minor arc is defined to be the measure of the central angle that intercepts the arc. The measure of a major arc is 360° minus the measure of the minor arc with the same endpoints.

In the figure above, the measure of $\widehat{AB}$ is 75° because that is the measure of its central angle. The measure of $\widehat{ACB}$ is 360° – 75° or 285°.

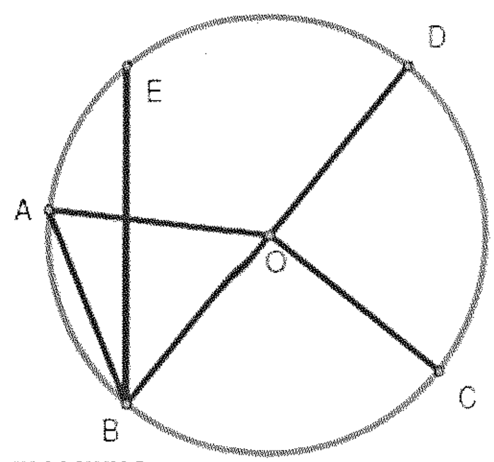
The **length** of an arc is different from its measure. The length is given in linear units and is determined as a portion of the length of the entire circumference of the circle. We will investigate the length of an arc in a later task. **Congruent arcs** have equal degree measures and equal lengths.

A **chord** is a *segment* whose endpoints lie on the circle. In the above figure, segment $\overline{AB}$ is a chord of ○*P*.

2. How many chords can be in a circle?

What is the longest chord in a circle?
Explain how you know?

3. Refer to the figure at the right.
Identify and name each of the following.
Be sure to use the correct notation.



a. Two different central angles
b. A minor arc
c. A major arc
d. A semicircle
e. Two different chords
f. The central angle subtended by $\widehat{AD}$

Use your protractor to help you find the following measures:
g. The measure of $\widehat{AC}$
h. The measure of $\widehat{DEC}$

4. Now it is time to use some of the terminology you have learned. Consider the following two theorems:

In the same circle or congruent circles, if two chords are congruent, their intercepted arcs are congruent.

In the same circle or congruent circles, if two arcs are congruent, then their chords are congruent.

a. Prove that each of the theorems is true.

b. Write the two theorems as one biconditional statement.

5. Use a compass to construct a circle on an unlined sheet of paper. Label the center of your circle.

a. Draw any chord, other than a diameter, on your circle.
Use your compass and a straightedge to construct a segment that represents the distance from the center of your circle to the chord. What is the relationship between the chord and the segment representing this distance?

b. Mary made the following conjecture: "If two chords of a circle are the same distance from the center of the circle, the chords are congruent." Mary is correct. Use what you learned in *Item 5a* to help convince Mary that her conjecture is correct.

c. State the converse of Mary's conjecture.

d. Write Mary's conjecture and its converse as a biconditional statement.

e. When a conjecture has been proven, it can be stated as a theorem. Write and illustrate this theorem in your *Circle Book.*

6. Ralph made the following conjecture: "A radius perpendicular to a chord bisects the chord."
a. Use your construction from *item 5a* to help convince yourself that Ralph's conjecture and the converse are true.

b. Write Ralph's conjecture and its converse as a biconditional statement and illustrate it in your *Circle Book.*

c. Ralph also believes that a radius perpendicular to a chord bisects the arc intercepted by the chord.

Is this true? How do you know?

7. Tevante examined his construction and his partner's construction. He believes that *any* line that is a perpendicular bisector of a chord of a circle must also contain the center of the circle. Is he right? How do you know?