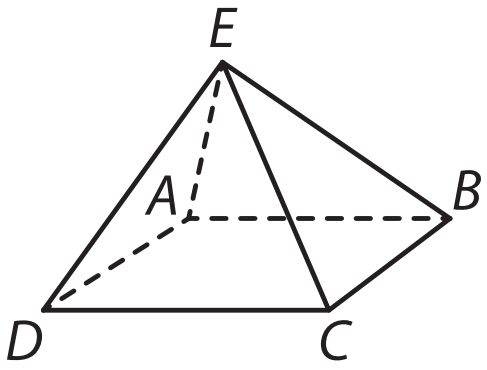
Lesson 1.5.2: Explaining ASA, SAS, and SSS

Warm-Up 1.5.2

The Great Pyramid of Giza is the oldest and largest of the three pyramids near Giza, Egypt. The pyramid has a square base and four congruent triangular faces.



1. Is it possible to determine if the sides of the triangular faces are congruent? Explain your reasoning.

2. Is it possible to determine if the angles of the triangular faces are congruent? Explain your reasoning.

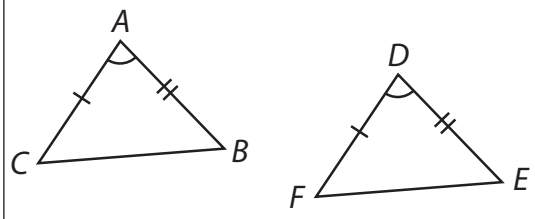
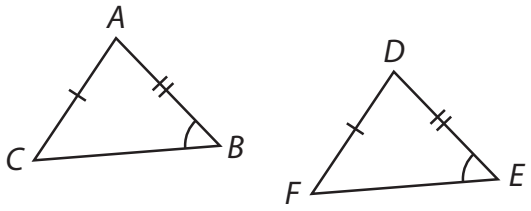
By looking at the information about each triangle, you can determine whether the triangles are congruent.

The **Side-Side-Side (SSS) Congruence Statement** states that if three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent.

If it is known that the corresponding sides are congruent, it is understood that the corresponding angles are also congruent.

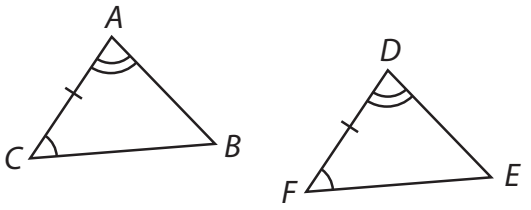
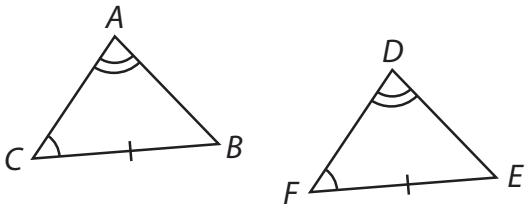
The **Side-Angle-Side (SAS) Congruence Statement** states that if two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

The **included angle** is the angle that is between the two congruent sides.

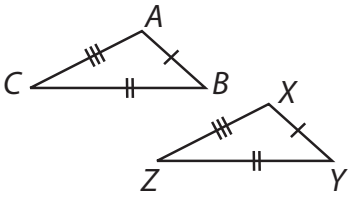
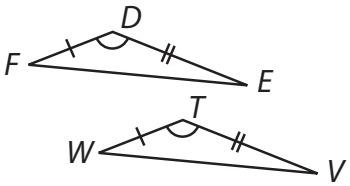
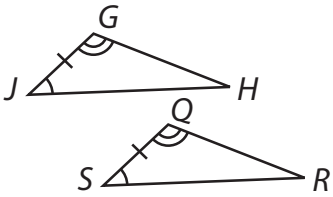
Included angle	Non-included angle
	
$\angle A$ is included between $\overline{CA}$ and $\overline{AB}$. $\angle D$ is included between $\overline{FD}$ and $\overline{DE}$.	$\angle B$ is NOT included between $\overline{CA}$ and $\overline{AB}$. $\angle E$ is NOT included between $\overline{FD}$ and $\overline{DE}$.

The **Angle-Side-Angle Congruence Statement,** or **ASA**, states that if two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

The **included side** is the side that is between the two congruent angles.

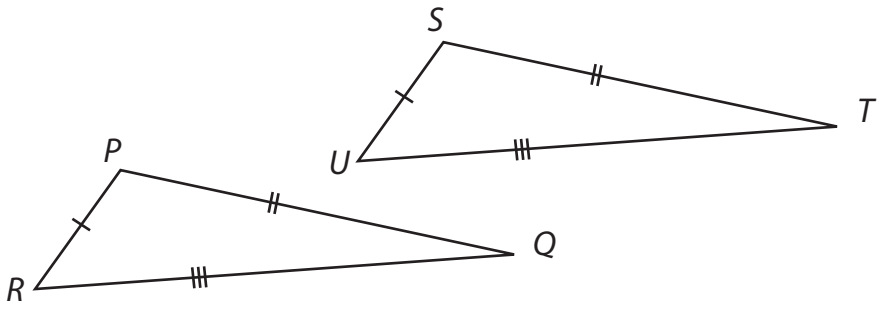
Included side	Non-included side
	
$\overline{AC}$ is included between $\angle C$ and $\angle A$. $\overline{FD}$ is included between $\angle F$ and $\angle D$.	$\overline{CB}$ is NOT included between $\angle C$ and $\angle A$. $\overline{FE}$ is NOT included between $\angle F$ and $\angle D$.

The following diagram compares these three congruence statements.

Side-Side-Side (SSS)	Side-Angle-Side (SAS)	Angle-Side-Angle (ASA)
		
$\triangle ABC \cong \triangle XYZ$	$\triangle DEF \cong \triangle TVW$	$\triangle GHJ \cong \triangle QRS$

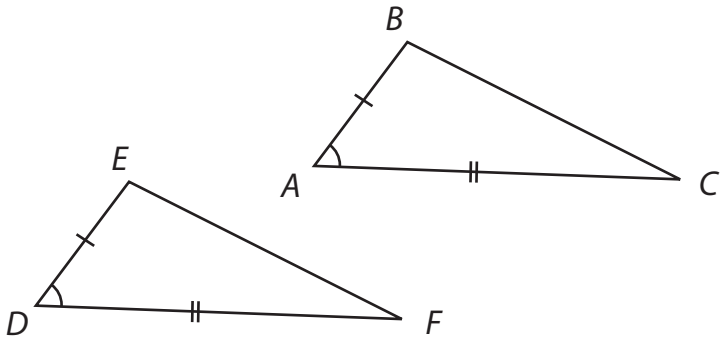
Example 1

Determine which congruence statement, if any, can be used to show that $\triangle PQR$ and $\triangle STU$ are congruent.



Example 2

Determine which congruence statement, if any, can be used to show that $\triangle ABC$ and $\triangle DEF$ are congruent.



Example 3

Determine which congruence statement, if any, can be used to show that $\triangle HIJ$ and $\triangle KLM$ are congruent if $\overline{HI} \cong \overline{KL}$, $\angle H \cong \angle K$, and $\angle I \cong \angle L$.

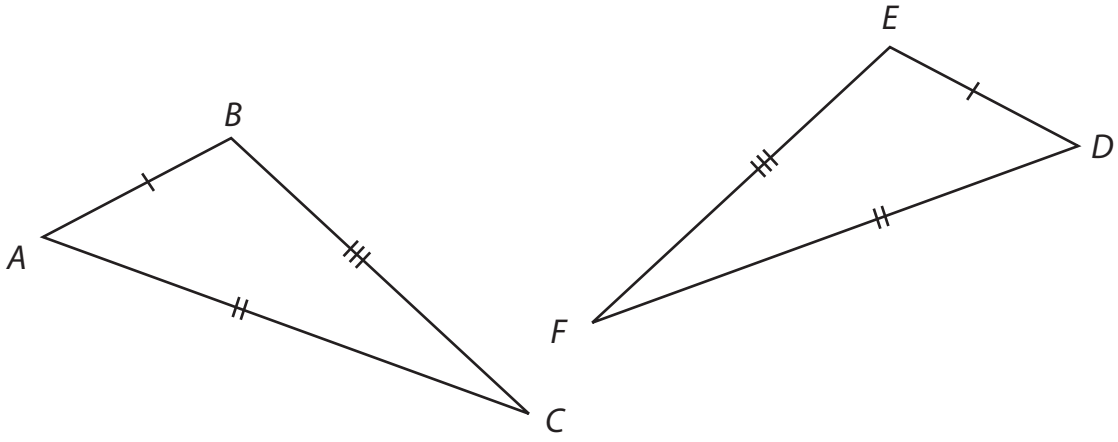
Example 4

Determine which congruence statement, if any, can be used to show that $\triangle PQR$ and $\triangle STU$ are congruent if $\overline{PQ} \cong \overline{ST}$, $\overline{PR} \cong \overline{SU}$, and $\angle Q \cong \angle T$.

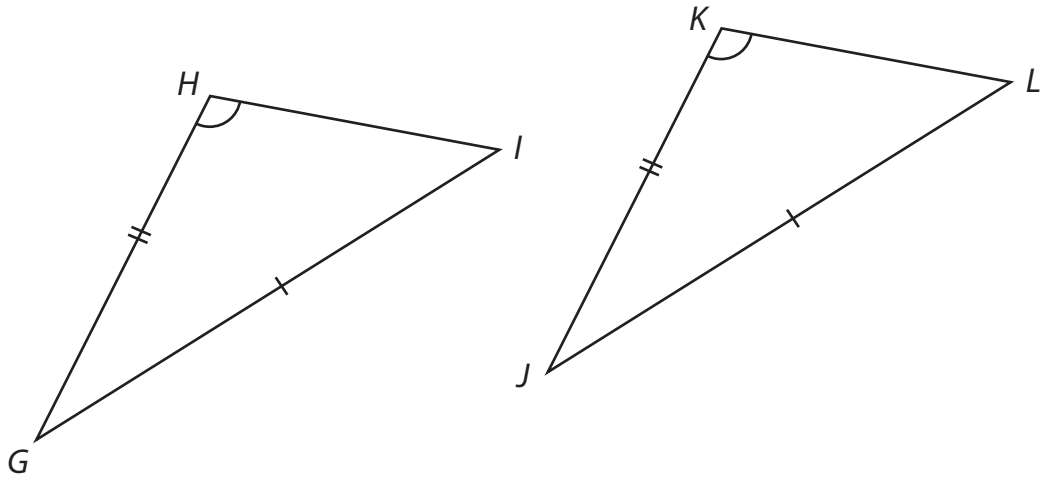
Practice 1.5.2: Explaining ASA, SAS, and SSS

For each diagram, determine which congruence statement can be used to show that the triangles are congruent. If it is not possible to prove triangle congruence, explain why not.

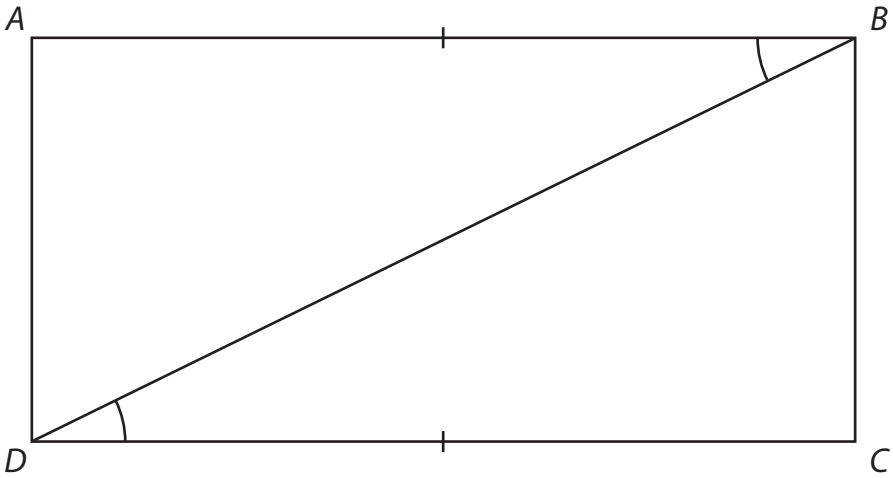
1.



2.



3. Based on the information in the diagram, is $\triangle ABD$ congruent to $\triangle CDB$?



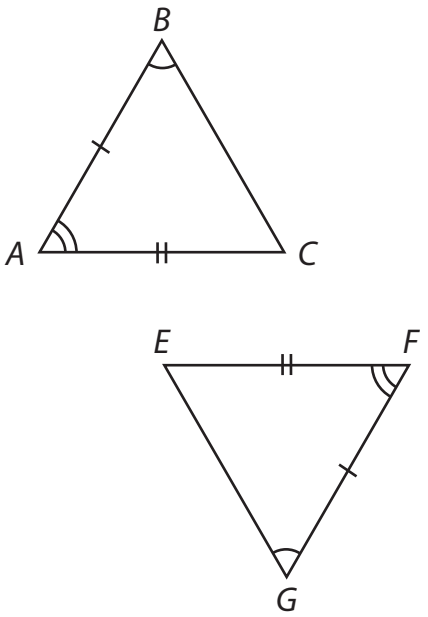
Use the given information to determine which congruence statement can be used to show that the triangles are congruent. If it is not possible to prove triangle congruence, explain why not.

4. $\triangle STU$ and $\triangle VWX$: $\angle S \cong \angle V$, $\angle T \cong \angle W$, and $\overline{ST} \cong \overline{VW}$

5. $\triangle MNO$ and $\triangle PQR$: $\angle O \cong \angle R$, $\overline{MO} \cong \overline{PR}$, and $\overline{NO} \cong \overline{QR}$

6. $\triangle GHI$ and $\triangle JKL$: $\angle G \cong \angle H$, $\overline{HI} \cong \overline{KL}$, $\angle J \cong \angle K$

7. Jessalyn found two vintage road signs at a thrift store. She is re-decorating her room and congruency is important for her decor. Based on the information about each sign shown in the diagram below, determine if the triangles are congruent. If so, name the congruent triangles and identify the congruence statement used.



8. Isaac needs two congruent sails for his sailboat. The boat supply shop has only two sails in stock. Based on the information about each sail, determine if the sails are congruent. If so, name the congruent triangles and identify the congruence statement used.