REVIEW PROBLEMS FOR INTERMEDIATE ALGEBRA ASSESSMENT TEST-Rev 1

1. Simplify and combine like terms: $-18x + 3(2x - 8) - 9x - (4 - 6x)$

2. Multiply: $(2z^3 - 4z^2 + 6z - 1)(3z + 2)$

3. Multiply: $(2a + 3)(4a^2 - 6a + 9)$

4. Multiply: $(5x - 2)(3x + 4)$

5. $3\sqrt{25} =$

6. $\dfrac{6^2 y^3 z^5}{6^4 y^2 z^7} =$

7. $\dfrac{5^0 (x^2)^3 y}{5x(y^2)^5} =$

8. $\dfrac{7a^3(b+c)^2}{14a(b+c)} =$

9. $3^{2^4} =$

10. $16^{\frac{5}{4}} =$

11. Simplify. All exponents must be positive.

$$\left(\frac{x^{-\frac{3}{5}} z^{-1}}{2^{-2} z^{-\frac{1}{2}} x}\right)^{-1} =$$

12. Solve: $7z - 3z + 12 - 5 = 8z + 7$

13. Solve: $-4(3 + 2m) - m = -3m$

14. Solve for x: $Q = 3x + 2y + 4z$

15. Solve for x: $4^x = 8^{x-3}$

16. Scott Hardy invested some money at 12% and $4000 less than this amount at 14%. Find the amount invested at each rate if his total annual interest income is $4,120.

17. A pharmacist needs 100 liters of a 50% alcohol solution. She has a 30% alcohol solution and an 80% alcohol solution that she can mix. How many liters of each does she need?

18. Two people need to sort a pile of bottles at the recycling center. Working alone, one person could do the entire job in 9 hours, and the other person could do the entire job in 6 hours. How long would it take them to complete the job if they work together?

19. Solve the system: $2x + 3y = 10$
$-3x + 2y = 11$

20. Solve the system: $3x + 4y = 8$
$6x = 7 - 8y$

21. Steve and Ross must paint Steve's entire house. Steve bought 3 gallons of paint and 2 brushes at one store for \$48. Ross bought 5 gallons of paint and 1 brush at another store for \$66. Assuming the price per gallon and cost per brush were the same at both stores, find what they paid per gallon for paint and the cost per brush.

22. Solve: $\frac{2q+1}{3} - \frac{q-1}{4} = -2$

23. Solve: $5x^2 - 3x = 0$

24. Find the roots of the following equation by factoring: $10x^2 - 13x = 3$

25. Find the roots of the following equation using the quadratic formula:

$2x^2 + 3x = 10$

26. Find the roots of the following equation: $(2x + 1)(x - 2) = -3$

27. Solve: $\frac{y}{3} - \frac{y}{7} = 1$

28. Solve: $\frac{1}{m+4} - \frac{3}{2m+8} = \frac{1}{2}$

29. Rationalize the denominator and simplify: $\frac{-6\sqrt{3}}{\sqrt{2}}$

30. Multiply and combine like terms: $(2x + 3)^2$

31. Multiply and combine like terms: $(2x + 3)^2 - (x + 2)^2$

32. Evaluate the following expression if $x = 3$ and $y = -2$.

$3x^2y - 4xy^2$

33. Find the distance between the point (-4, -2) and the point (6, -1).

34. Divide: $\dfrac{8m^4 - 6m^3 + 2m}{2m}$

35. Divide: $(15p^2 + 11p - 17) \div (3p - 2)$

36. What quadrant is the point (-3,10) in?

37. Reduce, if possible: $\dfrac{6x^2 + 7x - 5}{3x + 5}$

38. Multiply: $\dfrac{2p^2 - 5p - 12}{5p^2 - 18p - 8} \cdot \dfrac{25p^2 - 4}{30p - 12}$

39. Divide: $\dfrac{25p^3q^2}{8p^4q} \div \dfrac{15pq^2}{16p^5}$

40. Divide: $(y^2 + 2y) \div \dfrac{y + 5}{y^2 + 4y - 5}$

41. Combine: $\dfrac{3}{t - 2} - \dfrac{5}{2 - t}$

42. Combine: $\dfrac{2}{y + 1} + \dfrac{6}{y - 1}$

43. Combine: $\dfrac{3r}{10r^2 - 3rs - s^2} + \dfrac{2r}{2r^2 + rs - s^2}$

44. Simplify: $\dfrac{\frac{3}{x} - 5}{6 + \frac{1}{x}}$

45. Sketch the graph of the line $3x - y = 4$.

46. If the slope, m, of line is $\frac{1}{2}$, and the y- intercept is at (0, 3), what is the equation of the line? Express the answer in slope-intercept form.

47. What is the slope of the line between the two points (4, -3) and (-2, -1)?

48. What is the equation of the line with slope = $\frac{2}{3}$ and containing point (2, -1)? Express the answer in standard form.

49. What is the equation of the line which goes through the two points (3, 5) and (-1, 0)? Express the answer in standard form.

50. Sketch the graph of the line y = -2.

51. Solve $\sqrt{x + 2} = x - 4$

52. Solve $|x - 2| < 3$

53. 4! =

54. Solve $-3x > 12 + x$

SOLUTIONS FOR INTERMEDIATE ALGEBRA ASSESSMENT TEST

1. $-18x + 3(2x - 8) - 9x - (4 - 6x)$

   Use the distributive law to remove parentheses:

   $-18x + 6x - 24 - 9x - 4 + 6x$

   Combine like terms: $-15x - 28$

2. Multiply vertically to easily line up like terms:

$$\begin{array}{r} 2z^3 - 4z^2 + 6z - 1 \\ \underline{\quad\quad\quad\quad 3z + 2} \\ 6z^4 - 12z^3 + 18z^2 - 3z \quad\quad \\ \underline{4z^3 - 8z^2 + 12z - 2} \\ 6z^4 - 8z^3 + 10z^2 + 9z - 2 \end{array}$$

3. Multiply vertically as in #2.

$$\begin{array}{r} 4a^2 - 6a + 9 \\ \underline{2a + 3} \\ 8a^3 - 12a^2 + 18a \quad\quad \\ \underline{+ 12a^2 - 18a + 27} \\ 8a^3 \quad\quad\quad\quad\quad + 27 \end{array}$$

   answer: $8a^3 + 27$

4. Multiply using the acronym FOIL (firsts, outsides, insides, lasts)
   $(5x - 2)(3x + 4) = 15x^2 + 20x - 6x - 8$
   $= 15x^2 + 14x - 8$

5. $3^{\sqrt{25}} = 3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

6. When dividing, the base stays the same and you subtract exponents negative exponents then go to the denominator:

$$\frac{6^2 y^3 z^5}{6^4 y^2 z^7} = \frac{y}{6^2 z^2} = \frac{y}{36z^2}$$

7. When raising a power to a power, you multiply exponents. Any non-zero expression raised to the zero power is 1.

$$\frac{5^0(x^2)^3 y}{5x(y^2)^5} = \frac{1x^6 y}{5xy^{10}} = \frac{x^5}{5y^9}$$

8. $\frac{7a^3(b+c)^2}{14a(b+c)} = \frac{a^2(b+c)}{2}$

$\frac{(b+c)^2}{b+c}$ works exactly like $\frac{x^2}{x}$. Just as $\frac{x^2}{x}$ = x, $\frac{(b+c)^2}{b+c}$ = (b + c)^1 or (b + c)

9. $3^{2^4} = 3^{16}$

10. $16^{\frac{5}{4}}$ means the 4th root of 16 to the 5th power:

$$(\sqrt[4]{16})^5 = 2^5 = 32$$

11. Remember, to raise a power to a power, multiply exponents.

$$\left(\frac{x^{-\frac{3}{5}}z^{-1}}{2^{-2}z^{-\frac{1}{2}}x}\right)^{-1} = \frac{x^{\frac{3}{5}}z^1}{2^2 z^{\frac{1}{2}} x^{-1}}$$

The x^{-1} must go to the numerator and become x^1:

$$\frac{x^{\frac{3}{5}}x^1 z^1}{4z^{\frac{1}{2}}}$$

When multiplying, add exponents. When dividing, subtract exponents.

$$= \frac{x^{\frac{8}{5}}z^{\frac{1}{2}}}{4}$$

12. 7z – 3z + 12 – 5 = 8z + 7

Combine like terms:

4z + 7 = 8z + 7
Add –4z to both sides: 7 = 4z + 7
Add –7 to both sides: 0 = 4z
Divide both sides by 4: 0 = z

13. $-4(3 + 2m) - m = -3m$

Use the distributive law to remove parentheses:

$-12 - 8m - m = -3m$
$-12 - 9m = -3m$

Add 9m to both sides: $-12 = 6m$
Divide both sides by 6: $-2 = m$

14. $Q = 3x + 2y + 4z$

Add $-2y - 4z$ to both sides in order to isolate the term containing the unknown, x:

$Q - 2y - 4z = 3x$

Divide both sides by 3: $\dfrac{Q - 2y - 4z}{3} = x$

15. $4^x = 8^{x-3}$

Rewrite 4 and 8 as powers of 2: $(2^2)^x = (2^3)^{x-3}$
When raising a power to a power, you multiply exponents: $2^{2x} = 2^{3x-9}$
Hence, $2x = 3x - 9$
$-x = -9$
$x = 9$

16. Let x = the amount of money invested at 12%.

x – 4000 = the amount of money invested at 14%.

Then, $.12x + .14(x - 4000) = 4120$

Multiply each term by 100 to remove decimal points:

$12x + 14(x - 4000) = 412{,}000$
$12x + 14x - 56000 = 412{,}000$
$26x = 468{,}000$
$x = 18{,}000$
He invested \$18,000 at 12% and \$14,000 at 14%

17. Let x = amount of 30% alcohol solution.
y = amount of 80% alcohol solution.

Equate amount of solution: $x + y = 100$

Equate amount of alcohol: $.30x + .80y = .50(100)$
or $30x + 80y = 50(100)$

If $x + y = 100$, then $y = 100 - x$

$$30x + 80(100 - x) = 5000$$
$$30x + 8000 - 80x = 5000$$
$$-50x = -3000$$
$$x = 60$$

She needs 60 liters of the 30% solution and 40 liters of the 80% solution.

18. First person takes 9 hours.
Second person takes 6 hours.
Together they take x hours.

Equate what can be done in 1 hour.

$$\frac{1}{9} + \frac{1}{6} = \frac{1}{x}$$

Multiply both sides by the L. C. D. = 18x:

$$2x + 3x = 18$$
$$5x = 18$$

$$x = \frac{18}{5} \text{ or } 3\frac{3}{5}$$

Together it would take $3\frac{3}{5}$ hours.

19. $2x + 3y = 10$
$-3x + 2y = 11$

Multiply the top equation by 3 and the bottom by 2 so that the x terms will be eliminated.

$$6x + 9y = 30$$
$$\underline{-6x + 4y = 22}$$
$$13y = 52$$
$$y = 4$$

Substitute y = 4 in the first equation:
$2x + 3(4) = 10$
$2x + 12 = 10$
$2x = -2$
$x = -1$

20. $3x + 4y = 8$
$6x = 7 - 8y$

In the second equation add 8y to both sides to get it in standard form:

$$3x + 4y = 8$$
$$6x + 8y = 7$$

Multiply the top equation by –2:

$$\begin{array}{r} -6x - 8y = -16 \\ \underline{6x + 8y = \quad 7} \\ 0 = -9 \end{array}$$

That cannot happen. The lines are parallel and the system is inconsistent.

21. Let g = price for one gallon of paint
Let b = price for one brush

$$3g + 2b = 48$$
$$5g + b = 66$$

In the second equation: $b = 66 - 5g$. Substitute this into the top equation:

$$\begin{array}{r} 3g + 2(66 - 5g) = 48 \\ 3g + 132 - 10g = 48 \\ -7g + 132 = 48 \\ -7g = -84 \\ g = 12 \end{array}$$

$$b = 66 - 5g = 66 - 5(12) = 66 - 60 = 6$$

A gallon of pain costs \$12; one brush costs \$6.

22. $\frac{2q+1}{3} - \frac{q-1}{4} = -2$

Multiply both sides by 12 to eliminate the denominators:

$$\begin{array}{r} 4(2q + 1) - 3(q - 1) = -2(12) \\ 8q + 4 - 3q + 3 = -24 \\ 5q + 7 = -24 \\ 5q = -31 \end{array}$$

$$q = -\frac{31}{5}$$

23. $5x^2 - 3x = 0$

Factor out the common factor of x: $x(5x - 3) = 0$

The product of two factors is 0, and hence one of the factors is 0:

$x = 0$ or $5x - 3 = 0$

$5x = 3$

$x = \frac{3}{5}$

24. $10x^2 - 13x = 3$

Add –3 to both sides so that 0 is on one side of the equation:

$10x^2 - 13x - 3 = 0$

Factor the left hand side:

$(5x + 1 = 0)\ (2x - 3) = 0$

The product of two factors is 0, and hence one of the factors is 0:

$5x + 1 = 0$ or $2x - 3 = 0$

$5x = -1$ $\quad 2x = 3$

$x = -\frac{1}{5}$ $\quad x = \frac{3}{2}$

25. $2x^2 + 3x = 10$

Add –10 to both sides so that 0 is on one side of the equation:

$2x^2 + 3x - 10 = 0$

The quadratic formula says that in the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In the given equation, then, a = 2, b = 3, and c = -10

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-10)}}{2(2)} = \frac{-3 \pm \sqrt{9 + 80}}{4} = \frac{-3 \pm \sqrt{89}}{4}$$

26. $(2x + 1)(x - 2) = -3$

First multiply the 2 binomials:
$2x^2 - 3x - 2 = -3$

Add 3 to both sides:
$2x^2 - 3x + 1 = 0$

Factor: $(2x - 1)(x - 1) = 0$

$2x - 1 = 0$ or $x - 1 = 0$
$2x = 1$ $\quad x = 1$
$x = \frac{1}{2}$

27. $\frac{y}{3} - \frac{y}{7} = 1$
Multiply both sides by 21:
$7y - 3y = 21$
$4y = 21$
$y = \frac{21}{4}$

28. $\frac{1}{m+4} - \frac{3}{2m+8} = -\frac{1}{2}$

Factor the denominators, wherever possible:

$$\frac{1}{m+4} - \frac{3}{2(m+4)} = -\frac{1}{2}$$

Multiply both sides by the L. C. D. of 2(m + 4):

$1(2) - 3 = -1(m + 4)$
$2 - 3 = -m - 4$
$3 = -m$
$-3 = m$

29. $\frac{-6\sqrt{3}}{\sqrt{2}}$

Multiply numerator and denominator by $\sqrt{2}$:

$$\frac{-6\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-6\sqrt{6}}{2} = -3\sqrt{6}$$

30. $(2x + 3)^2 = (2x + 3)(2x + 3) = 4x^2 + 6x + 6x + 9$
$= 4x^2 + 12x + 9$

31. $(2x + 3)^2 - (x + 2)^2$

From problem #30 $(2x + 3)^2 = 4x^2 + 12x + 9$

Then, $(x + 2)^2 = (x + 2)(x + 2) = x^2 + 2x + 2x + 4 = x^2 + 4x + 4$

$(2x + 3)^2 – (x + 2)^2 = 4x^2 + 12x + 9 – (x^2 + 4x + 4)$
$= 4x^2 + 12x + 9 – x^2 – 4x – 4$
$= 3x^2 + 8x + 5$

32. $3x^2y – 4xy^2$

If x = 3 and y = -2, then the problem becomes:

$3(3)^2 (-2) – 4(3)(-2)^2 =$
$3(9)(-2) – 4(3)(4) =$
$-54 – 48 = -102$

33. The distance between points (x_1, y_1) and (x_2, y_2) is given by the formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
The given points are (-4, -2) and (6, -1)
$x_1 = -4; \quad x_2 = 6$
$y_1 = -2; \quad y_2 = -1$
Notice that $x_2 - x_1 = 6 - (-4) = 6 + 4 = 10$
$y_2 - y_1 = -1 - (-2) = -1 + 2 = 1$

Therefore, $d = \sqrt{10^2 + 1^2} = \sqrt{100 + 1} = \sqrt{101}$

34. $\dfrac{8m^4 - 6m^3 + 2m}{2m} = \dfrac{8m^4}{2m} - \dfrac{6m^3}{2m} + \dfrac{2m}{2m} = 4m^3 - 3m^2 + 1$

35.

$$
\begin{array}{r l}
 & 5p + 7 + \dfrac{-3}{3p - 2} \\
3p-2 & \overline{)15p^2 + 11p - 17} \\
 & 15p^2 - 10p \\
\hline
 & \quad 21p - 17 \\
 & \quad \underline{21p - 14} \\
 & \qquad -3
\end{array}
$$

36. Quadrant II

37. $$\frac{6x^2+7x-5}{3x+5}=\frac{(3x+5)(2x-1)}{3x+5}=2x-1$$

38. $$\frac{2p^2-5p-12}{5p^2-18p-8}\bullet\frac{25p^2-4}{30p-12}=\frac{(2p+3)(p-4)}{(5p+2)(p-4)}\bullet\frac{(5p-2)(5p+2)}{6(5p-2)}=\frac{2p+3}{6}$$

39. $$\frac{25p^3q^2}{8p^4q}\div\frac{15pq^2}{16p^5}$$

To divide fraction, invert the divisor and multiply:

5 p^2 2   2 p

$$\frac{25p^3q^2}{8p^4q}\bullet\frac{16p^5}{15pq^2}=\frac{10p^3}{3q}$$

3

40. To divide fractions, invert the divisor and multiply:

$$(y^2+2y)\div\frac{y+5}{y^2+4y-5}=(y^2+2y)\bullet\frac{y^2+4y-5}{y+5}$$

$$=y(y+2)\bullet\frac{(y+5)(y-1)}{y+5}=y(y+2)(y-1)$$

41. $$\frac{3}{t-2}-\frac{5}{2-t}$$

Note that 2 - t = (-1)(t - 2). The problem can then be written:

$$\frac{3}{t-2}-\frac{5}{(-1)(t-2)}$$

The two negatives become positive: $=\frac{3}{t-2}+\frac{5}{t-2}=\frac{8}{t-2}$

42. $$\frac{2}{y+1}+\frac{6}{y-1}$$

The L. C. D. is (y + 1)(y – 1): $\frac{2(y-1)}{(y+1)(y-1)}+\frac{6(y+1)}{(y+1)(y-1)}$

$$=\frac{2y-2+6y+6}{(y+1)(y-1)}=\frac{8y+4}{(y+1)(y-1)}\ or\ \frac{4(2y+1)}{(y+1)(y-1)}$$

43. $$\frac{3r}{10r^2 - 3rs - s^2} + \frac{2r}{2r^2 + rs - s^2}$$

Factor the denominators:

$$\frac{3r}{(5r + s)(2r - s)} + \frac{2r}{(2r - s)(r + s)}$$

The L. C. D. is (5r + s)(2r - s)(r + s)

$$\frac{3r(r + s)}{(5r + s)(2r - s)(r + s)} + \frac{2r(5r + s)}{(2r - s)(r + s)(5r + s)}$$

$$= \frac{3r^2 + 3rs + 10r^2 + 2rs}{(5r + s)(2r - s)(r + s)} = \frac{13r^2 + 5rs}{(5r + s)(2r - s)(r + s)}$$

44. $$\frac{\frac{3}{x} - 5}{6 + \frac{1}{x}}$$

In the numerator, the L. C. D. is x and it becomes:

$$\frac{3 - 5x}{x}$$

Similarly, the denominator is:

$$\frac{6x + 1}{x}$$

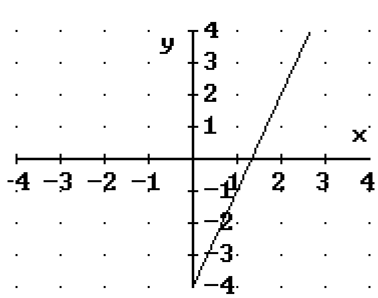
Invert the denominator, and multiply:

$$\frac{3 - 5x}{x} \bullet \frac{x}{6x + 1} = \frac{3 - 5x}{6x + 1}$$

45. 3x – y = 4

One way to sketch this is to plot 3 points and then connect the points.

1) If x = 0, then y = -4.
2) If x = 1, then y = -1.
3) If x = 2, then y = 2.



46. Slope-intercept form is y = mx + b where m is the slope and b is the y-intercept. This problem states that the slope is $\frac{1}{2}$ and the y-intercept in at (0, 3). Hence, the equation is:

$$y = \frac{1}{2}x + 3$$

47. Slope is defined to be:

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Given the point (4, -3) and (-2, -1):

$$m = \frac{-3 - (-1)}{4 - (-2)} = \frac{-3 + 1}{4 + 2} = \frac{-2}{6} = \frac{-1}{3}$$

48. The point-slope form of an equation is:

$m(x - x_1) = y - y_1$

Given m = $\frac{2}{3}$ and the point = (2, -1), the equation is:

$$\frac{2}{3}(x - 2) = y - (-1)$$

$$\frac{2}{3}x - \frac{4}{3} = y + 1$$

Multiply by 3:

$$2x - 4 = 3y + 3$$

$$2x - 3y = 7$$

49. Pts: (3, 5) and (-1, 0) = The slope, m, = $\frac{5 - 0}{3 - (-1)} = \frac{5}{3 + 1} = \frac{5}{4}$

Using point (-1, 0) and the point-slope form from problem 48:

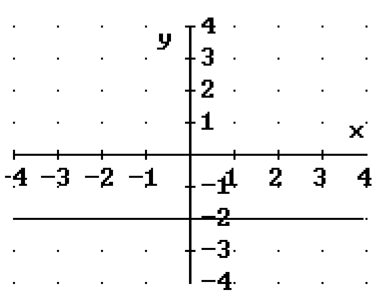
$$\frac{5}{4}(x + 1) = y - 0$$

$$\frac{5}{4}x + \frac{5}{4} = y$$

5x + 5 = 4y

5x – 4y = -5

50. Since y is always equal to –2, the graph is a horizontal line 2 units below the x-axis.



51. Solve $\sqrt{x+2} = x - 4$ Since a square root is involved, square both sides:

$(\sqrt{x+2})^2 = (x-4)^2$ Then you get $x + 2 = x^2 - 8x + 16$. Set = 0 getting $x^2 - 9x + 14 = 0$. Factor the quadratic getting (x-7)(x-2) = 0. Solving you get x = 7 or x = 2. 7 checks but 2 does not, so the answer is x = 7.

52. Solve $|x-2| < 3$ For absolute value with less than, there are two cases: $x-2 < 3$ and $x-2 > -3$. Solving the first gives $x < 5$; solving the second gives $x > -1$. Therefore $-1 < x < 5$

53. 4! = 4 x 3 x 2 x 1 = 24

54. $-3x > 12 + x$ Subtract x from both sides gives $-4x > 12$. Divide both sides by -4 giving $x < -3$. Remember, when you multiply or divide an inequality by a negative, the inequality reverses.