Guided Practice 3.1.6

Example 1

Solve $x^2 - 8x + 16 = 4$.

1. Determine if $x^2 - 8x + 16$ is a perfect square trinomial.

   Take one-half of the value of b and then square the result. If this is equal to the value of c, then the expression is a perfect square trinomial.

   $$\left(\frac{b}{2}\right)^2 = \left(\frac{-8}{2}\right)^2 = 16$$

   $x^2 - 8x + 16$ is a perfect square trinomial because the square of one-half of –8 is 16.

2. Write the left side of the equation as a binomial squared.

   One-half of b is –4, so the left side of the equation can be written as $(x - 4)^2$.

   $$(x - 4)^2 = 4$$

3. Take the square root of both sides of the equation to solve for x.

$(x - 4)^2 = 4$	Perfect square trinomial
$x - 4 = \pm 2$	Take the square root of both sides.
$x = 4 \pm 2$	Add 4 to both sides.
$x = 4 + 2 = 6$ or $x = 4 - 2 = 2$	Separate the result into two equations and solve for x.

4. Determine the solution(s).

   The equation has two solutions, $x = 2$ and $x = 6$.

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Example 2

Solve $x^2 + 6x + 4 = 0$ by completing the square.

1. Determine if $x^2 + 6x + 4$ is a perfect square trinomial.

   Take one-half of the value of b and then square the result. If this is equal to the value of c, then the expression is a perfect square trinomial.

   $$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$$

   $x^2 + 6x + 4$ is not a perfect square trinomial because the square of one-half of 6 is not 4.

2. Complete the square.

$x^2 + 6x + 4 = 0$	Original equation
$x^2 + 6x = -4$	Subtract 4 from both sides.
$x^2 + 6x + 3^2 = -4 + 3^2$	Add the square of one-half of b to both sides of the equation to complete the square.
$x^2 + 6x + 9 = 5$	Simplify.

3. Express the perfect square trinomial as the square of a binomial.

   One-half of b is 3, so the left side of the equation can be written as $(x + 3)^2$.

   $(x + 3)^2 = 5$

4. Take the square root of both sides of the equation to solve for x.

$(x + 3)^2 = 5$	Equation from the previous step
$x+3=\pm\sqrt{5}$	Take the square root of both sides.
$x=-3\pm\sqrt{5}$	Subtract 3 from both sides.

5. Determine the solution(s).

   The equation $x^2 + 6x + 4 = 0$ has two solutions, $x = -3\pm\sqrt{5}$.

Example 3

Solve $5x^2 - 50x - 120 = 0$ by completing the square.

1. Determine if $5x^2 - 50x - 120 = 0$ is a perfect square trinomial.

   The leading coefficient is not 1.

   First divide both sides of the equation by 5 so that $a = 1$.

$5x^2 - 50x - 120 = 0$	Original equation
$x^2 - 10x - 24 = 0$	Divide both sides by 5.

   Now that the leading coefficient is 1, take one-half of the value of b and then square the result. If the expression is equal to the value of c, then the quadratic expression is a perfect square trinomial.

$$\left(\frac{b}{2}\right)^2=\left(\frac{-10}{2}\right)^2=25$$

   $x^2 - 10x - 24$ is not a perfect square trinomial because the square of one-half of -10 is 25, not -24.

2. Complete the square.

$x^2 - 10x - 24 = 0$	Equation from the previous step
$x^2 - 10x = 24$	Add 24 to both sides.
$x^2 - 10x + (-5)^2 = 24 + (-5)^2$	Add the square of one-half of *b* to both sides of the equation to complete the square.
$x^2 - 10x + 25 = 49$	Simplify.

3. Express the perfect square trinomial as the square of a binomial.

One-half of *b* is –5, so the left side of the equation can be written as $(x - 5)^2$.

$(x - 5)^2 = 49$

4. Isolate *x*.

$(x - 5)^2 = 49$	Equation
$x - 5 = \pm\sqrt{49} = \pm 7$	Take the square root of both sides.
$x = 5 \pm 7$	Add 5 to both sides.
$x = 5 + 7 = 12$ or $x = 5 - 7 = -2$	Separate the result into two equations and solve for *x*.

5. Determine the solution(s).

The equation $5x^2 - 50x - 120 = 0$ has two solutions, $x = -2$ and $x = 12$.