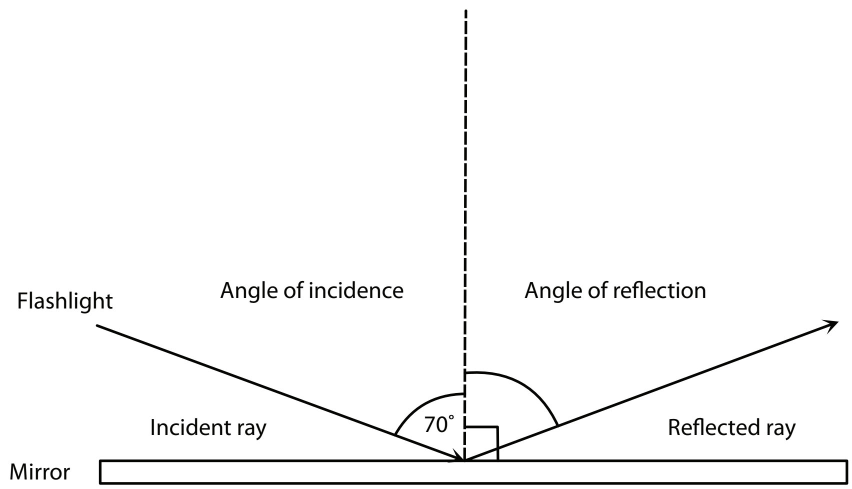
Lesson 1.9.1: Proving the Interior Angle Sum Theorem

Warm-Up 1.9.1

When a beam of light is reflected from a flat surface, the angle of incidence is congruent to the angle of reflection. The diagram below shows a ray of light from a flashlight being reflected off a mirror. Use the diagram to answer the questions that follow.



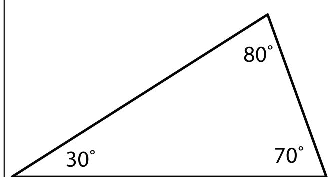
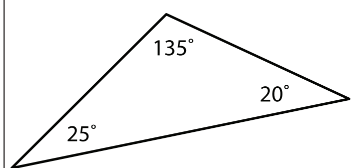
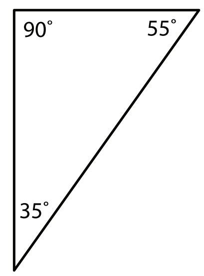
1. What is the measure of the angle of reflection? Explain how you found your answer.

2. What is the measure of the angle created by the mirror and the flashlight? Explain how you found your answer.

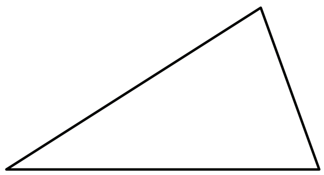
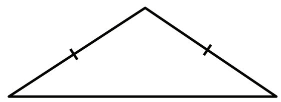
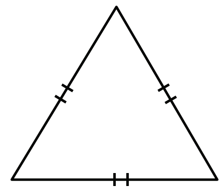
3. Describe how to determine the measure of the angle created by the mirror and the reflected ray of light.

Key Concepts

- There is more to a triangle than just three sides and three angles.
- Triangles can be classified by their angle measures or by their side lengths.
- Triangles classified by their angle measures can be acute, obtuse, or right triangles.
- All of the angles of an **acute triangle** are acute, or less than 90˚.
- One angle of an **obtuse triangle** is obtuse, or greater than 90˚.
- A **right triangle** has one angle that measures 90˚.

Acute triangle	Obtuse triangle	Right triangle
		
All angles are less than 90˚.	One angle is greater than 90˚.	One angle measures 90˚.

- Triangles classified by the number of congruent sides can be scalene, isosceles, or equilateral.
- A **scalene triangle** has no congruent sides.
- An **isosceles triangle** has at least two congruent sides.
- An equilateral triangle has three congruent sides.

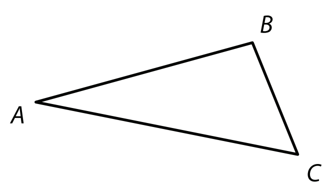
Scalene triangle	**Isosceles triangle**	**Equilateral triangle**
 No congruent sides	 At least two congruent sides	 Three congruent sides

- It is possible to create many different triangles, but the sum of the angle measures of every triangle is 180°. This is known as the Triangle Sum Theorem.

Theorem

Triangle Sum Theorem

The sum of the angle measures of a triangle is 180°.

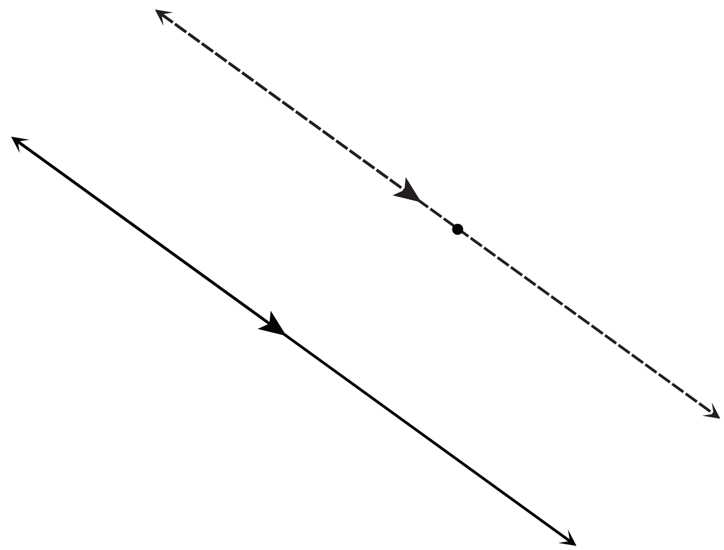


$m\angle A + m\angle B + m\angle C = 180$

- The Triangle Sum Theorem can be proven using the Parallel Postulate.
- The Parallel Postulate states that if a line can be created through a point not on a given line, then that line will be parallel to the given line.

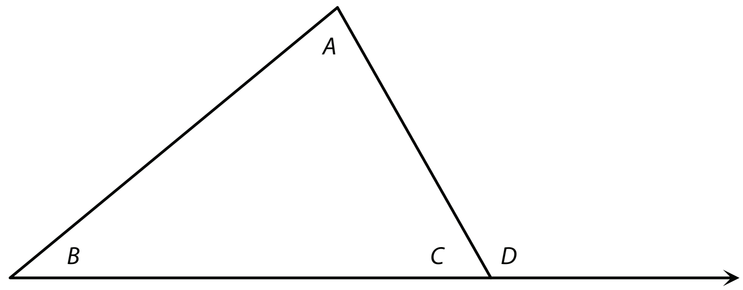
- This postulate allows us to create a line parallel to one side of a triangle to prove angle relationships.

Postulate
Parallel Postulate
Given a line and a point not on it, there exists one and only one straight line that passes through that point and never intersects the first line.

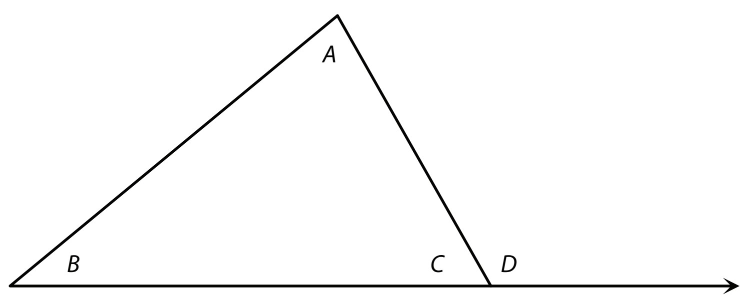


- This theorem can be used to determine a missing angle measure by subtracting the known measures from 180˚.
- Most often, triangles are described by what is known as the **interior angles** of triangles (the angles formed by two sides of the triangle), but exterior angles also exist.
- In other words, interior angles are the angles inside the triangle.
- **Exterior angles** are angles formed by one side of the triangle and the extension of another side.
- The interior angles that are not adjacent to the exterior angle are called the **remote interior angles** of the exterior angle.

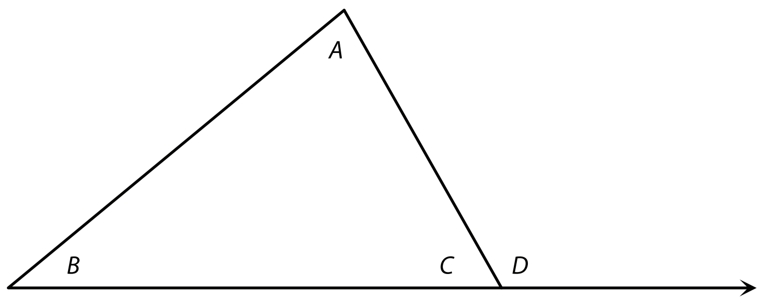
- The following illustration shows the differences among interior angles, exterior angles, and remote interior angles.



  - Interior angles: $\angle A$, $\angle B$, and $\angle C$
  - Exterior angle: $\angle D$
  - Remote interior angles of $\angle D$: $\angle A$ and $\angle B$
- Notice that $\angle C$ and $\angle D$ are supplementary; that is, together they create a line and sum to 180°.
- The measure of an exterior angle is equal to the sum of the measure of its remote interior angles. This is known as the Exterior Angle Theorem.

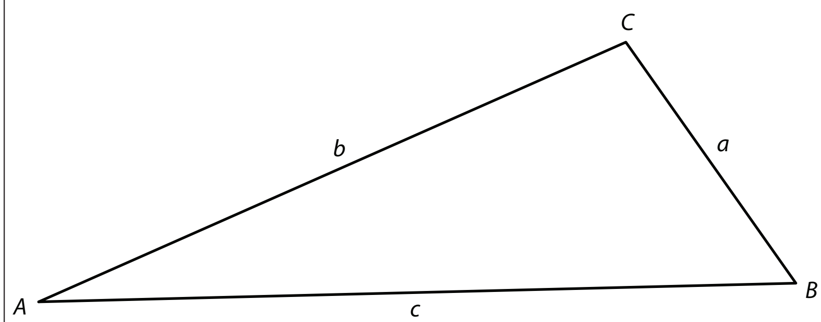
Theorem
Exterior Angle Theorem The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.  $m\angle D = m\angle A + m\angle B$

- This theorem can also be used to determine a missing angle measure of a triangle.
- The measure of an exterior angle will always be greater than either of the remote interior angles. This is known as the Exterior Angle Inequality Theorem.

Theorem
Exterior Angle Inequality Theorem
If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles.

$m\angle D > m\angle A$
$m\angle D > m\angle B$

- The following theorems are also helpful when finding the measures of missing angles and side lengths.

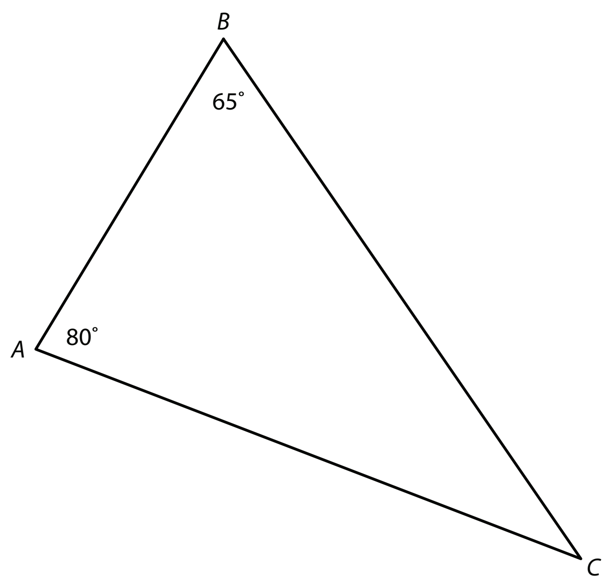
Theorem
If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

Theorem
If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.  $m\angle A < m\angle B < m\angle C$ $a < b < c$

- The Triangle Sum Theorem and the Exterior Angle Theorem will be proven in this lesson.

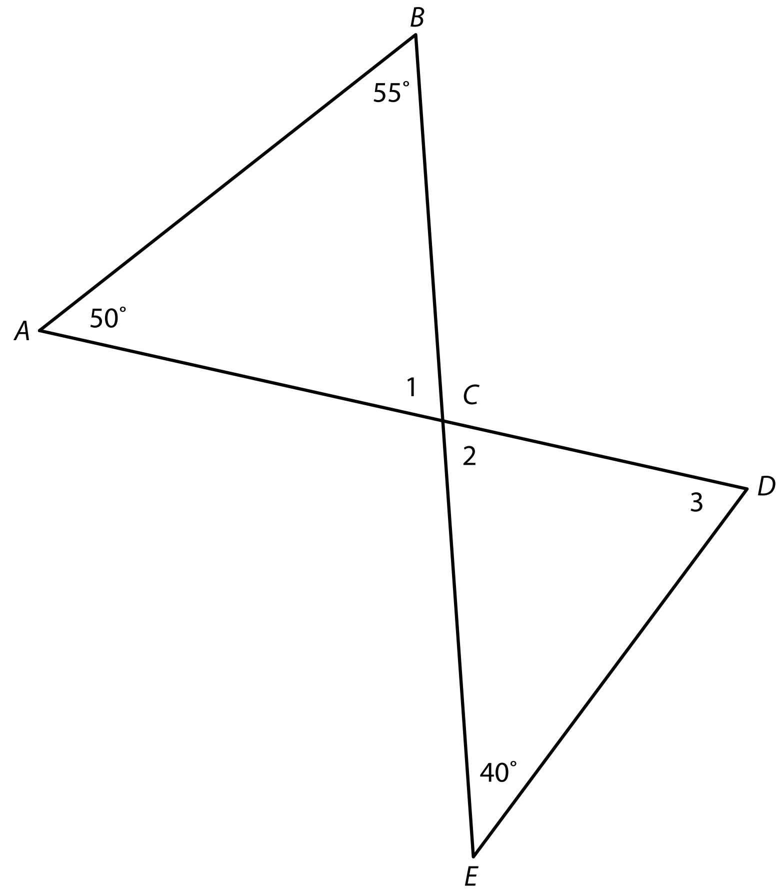
Example 1

Find the measure of $\angle C$.



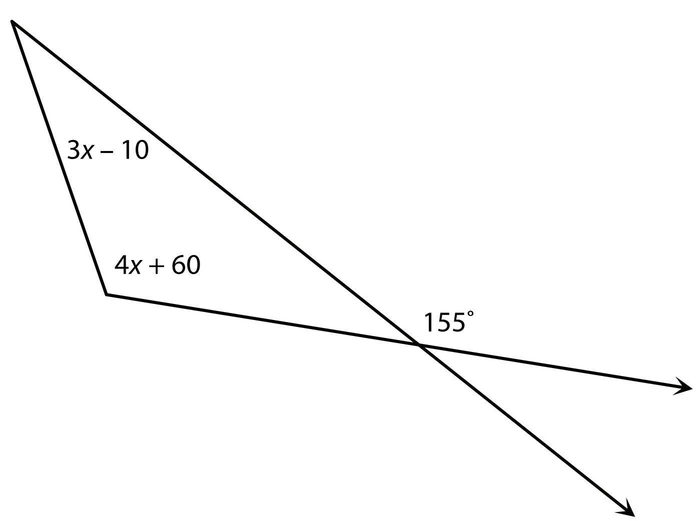
Example 2

Find the missing angle measures.



Example 3

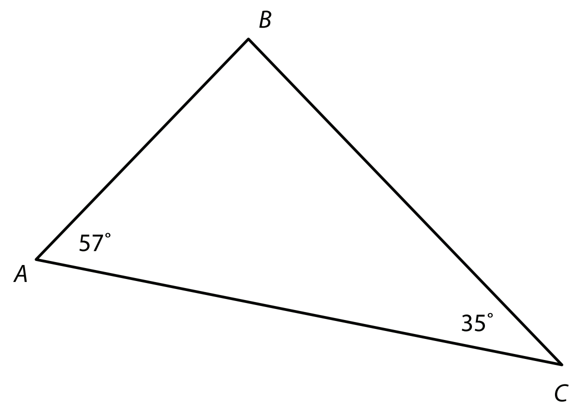
Find the missing angle measures.



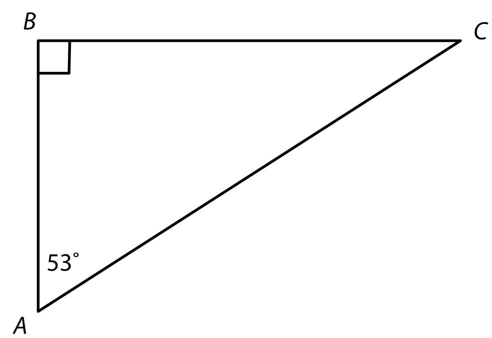
Practice 1.9.1: Proving the Interior Angle Sum Theorem

Use what you know about the sums of the interior and exterior angles of triangles to determine the measure of each identified angle.

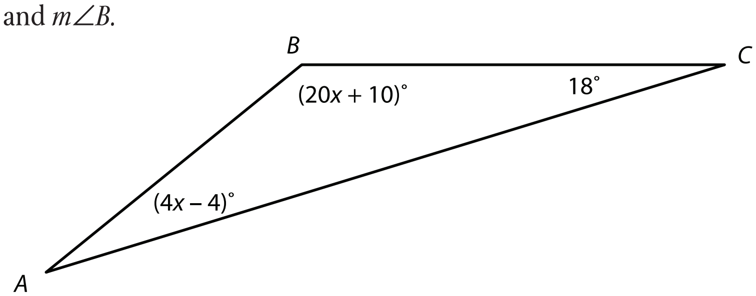
1. Find $m\angle B$.



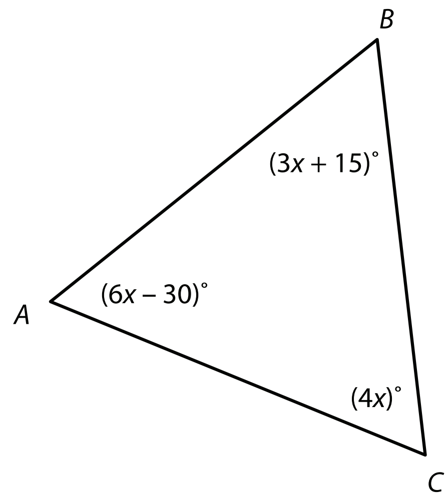
2. Find $m\angle C$.



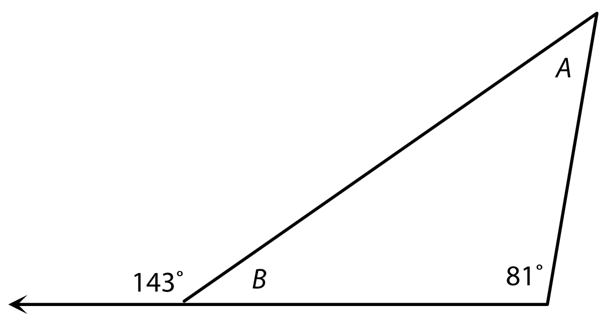
3. Find $m\angle A$ and $m\angle B$.



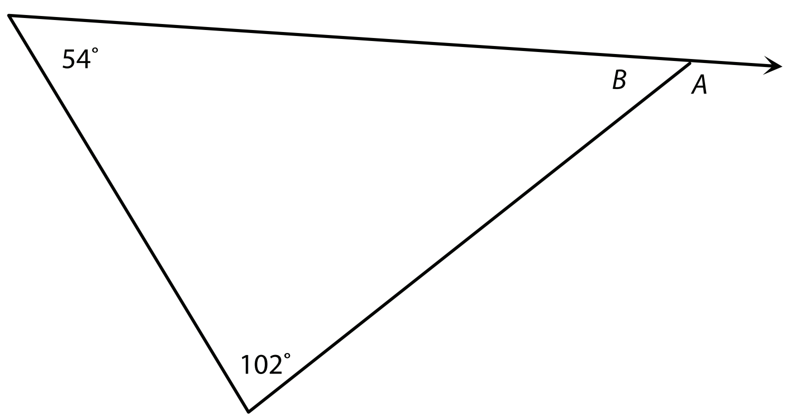
4. Find $m\angle A$, $m\angle B$, and $m\angle C$.



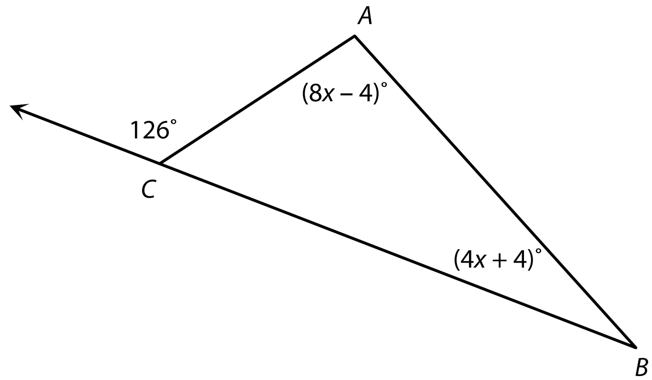
5. Find $m\angle A$ and $m\angle B$.



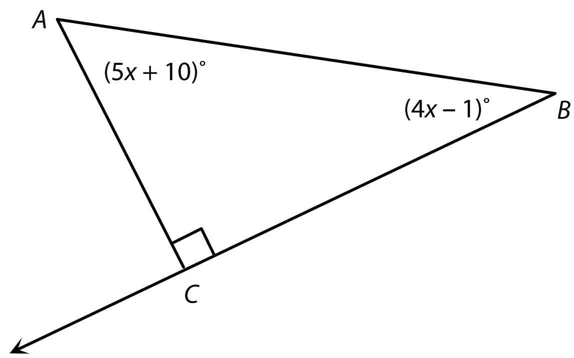
6. Find $m\angle A$ and $m\angle B$.



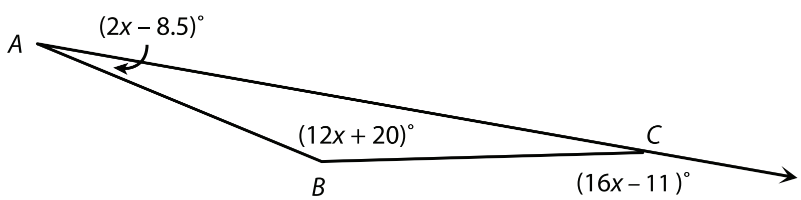
7. Find $m\angle CAB$ and $m\angle ABC$.



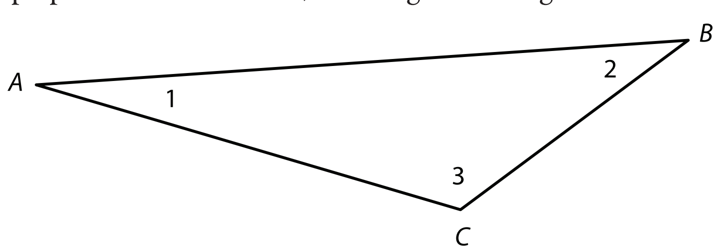
8. Find $m\angle CAB$ and $m\angle ABC$.



9. Find $m\angle CAB$ and $m\angle ABC$.

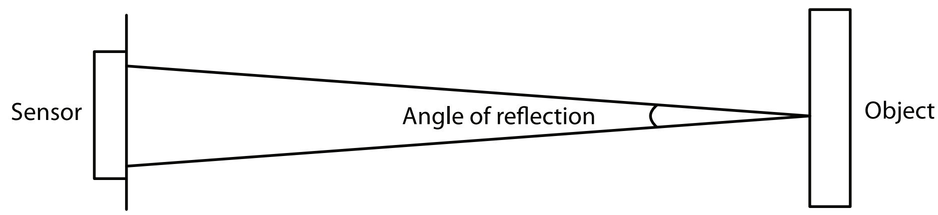


10. The Triangle Sum Theorem states that the sum of the angle measures of a triangle is 180°. Write a paragraph proof of this theorem, referring to the diagram below.



Problem-Based Task 1.9.1: Sensing Distance

Distance-measuring sensors frequently used in robotics send out a beam of infrared light that hits an object. The beam bounces off the object and returns to the sensor's detector, creating a triangle similar to the one below.



The angles of the triangle vary depending on the sensor's distance from the object. The sensor uses the angles to determine how far away the object is. As the angle of reflection increases, the calculated distance becomes more accurate. In each diagram below, the sensor is parallel to the object. Which of the sensors calculates a more accurate distance: Sensor A, or Sensor B? Explain your reasoning.